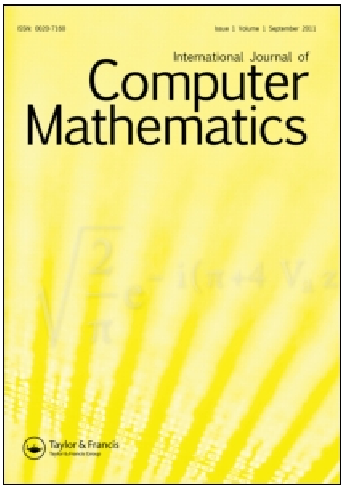

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Numerical simulation of blow-up solutions for the generalized Davey-Stewartson system

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Numerical simulation of blow-up solutions for the generalized Davey–Stewartson system

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Blow-up solutions for the generalized Davey–Stewartson system are studied numerically by using a split-step Fourier method. The numerical method has spectral-order accuracy in space and first-order accuracy in time. To evaluate the ability of the split-step Fourier method to detect blow-up, numerical simulations are conducted for several test problems, and the numerical results are compared with the analytical results available in the literature. Good agreement between the numerical and analytical results is observed.

Keywords: split-step Fourier method; generalized Davey–Stewartson system; blow-up

2010 AMS Subject Classification: 35B44; 35Q55; 65M70

1. Introduction

In this study, we present numerical simulations of blow-up solutions of the generalized Davey–Stewartson (GDS) system given by

$$iu_t + \sigma u_{xx} + u_{yy} = \kappa |u|^2 u + \gamma(\phi_x + \psi_y)u \tag{1}$$

$$\phi_{xx} + m_2\phi_{yy} + n\psi_{xy} = (|u|^2)_x \tag{2}$$

$$\lambda\psi_{xx} + m_1\psi_{yy} + n\phi_{xy} = (|u|^2)_y \tag{3}$$

where u and ϕ, ψ are, respectively, the complex- and the real-valued functions of spatial coordinates x, y and the time t. The parameters $\sigma, \kappa, \gamma, m_1, m_2, \lambda, n$ are real constants and σ is normalized as $|\sigma| = 1$. The parametric relation

$$(\lambda - 1)(m_2 - m_1) = n^2 \tag{4}$$

follows from the structure of the physical constants and plays a key role in the analysis of Equations (1)–(3). The GDS system has been derived to model $2+1$ dimensional wave propagation in a bulk medium composed of an elastic material with couple stresses [1]. When the parametric

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relation $n = 1 - \lambda = m_1 - m_2$ is satisfied, the GDS system reduces to the Davey–Stewartson (DS) system which consist of a coupled pair of equations

$$iu_t + \sigma u_{xx} + u_{yy} = \kappa |u|^2 u + \gamma u \phi_x$$
$$\phi_{xx} + m\phi_{yy} = (|u|^2)_x.$$

The DS system describes the evolution of water surface waves in the presence of gravity and capillarity [5]. The GDS system is classified according to the signs of parameters $(\sigma, m_1, m_2, \lambda)$ [2]. In the present study, we consider (–, +, +, +) hyperbolic–elliptic–elliptic (HEE) and (+, +, +, +) elliptic–elliptic–elliptic (EEE) cases. For these two cases, it has been proved in [2,6,7] that, at certain parameter values, initial data with negative energy can lead to singularity formation resulting in finite time blow-up of solutions of the GDS system. Note that the boundary conditions on the functions ϕ and ψ depend on the signs of m_1, m_2 and λ or, equivalently, on the types of the second and third equations of the GDS system (Equations (2) and (3)). For the HEE and EEE cases, Equations (2) and (3) are both elliptic type and ϕ and ψ are supposed to vanish at infinity. For all the other cases of the classification, either one or both of Equations (2) and (3) will always be hyperbolic type and ϕ and/or ψ will satisfy radiation conditions at infinity (which implies that the boundary conditions should be imposed along the characteristic lines). In this respect, the HEE and EEE cases are significantly simpler than the remaining cases of the GDS system. The literature on the DS system shows that there are no complete results about global existence and non-existence of solutions even for the elliptic-hyperbolic case of the DS system.

The purpose of this study is to develop a split-step Fourier method for the GDS system to detect the blow-up phenomena and estimate blow-up time. The essential ideas behind split-step Fourier methods are to decompose the initial and boundary value problem into subproblems which are simpler than the original problem and to expand the dependent variables in Fourier series. For a comprehensive list of general references on split-step Fourier methods, we refer to the review paper [9]. In the case of smooth solutions, split-step Fourier methods have previously been used to solve various equations of nonlinear dispersive wave propagation (see, e.g. [4,11–14] and the references therein). In the case of singularity formation (blow-up), this class of numerical methods seems to have attracted little attention for nonlinear dispersive equations, in general. For the DS system, to the best of the authors' knowledge, only two studies have been published on the numerical demonstration of blow-up via split-step Fourier methods [3,8]. The analyses used in these studies are based on the splitting scheme proposed in [14]. The convergence of the split-step Fourier method used for DS system has been demonstrated in [3]. The split-step method of the present study is based on an extension of the numerical scheme introduced in [14] for the DS system to the GDS system. We first split the initial and boundary value problem defined for the GDS system into linear and nonlinear subproblems that take into account dispersive effects and nonlinear effects, respectively. Next, the subproblems are solved using a Fourier pseudospectral approach, wherein all the nonlinear terms are evaluated in real space. We then approximate the solution of the original problem with a first-order accurate splitting. The novel aspect of the present method proposed for the GDS system lies in the fact that each subproblem is explicitly integrable. This fact makes the present algorithm extremely efficient, stable and fast. Finally, for both the HEE and EEE cases, we present numerical results for several test problems that have known analytical results. We observe that in both cases, our numerical results are in good agreement with the analytical ones. This good agreement indicates that the present numerical method accurately captures finite time blow-up, with a quantitative estimate of the blow-up time.

The paper is organized as follows. In the rest of Section 1, the most relevant properties of the GDS system are summarized. In Section 2, it is shown how the split-step method can be reformulated for the GDS system. In Section 3, the split-step method is tested for various blow-up problems of the GDS system. In Section 4, some conclusions are contained.

1.1 *Conserved quantities for the GDS system*

We now briefly mention some of the relevant properties of the GDS system. Assume that u, ϕ, ψ and all their derivatives converge to zero sufficiently rapidly at large spatial distances. Solutions of the GDS system subjected to these boundary conditions satisfy some conservation laws which imply that the conserved quantities

$$\begin{aligned}
\mathcal{N} &= \int_{R^2} |u|^2 \,\mathrm{d}x\,\mathrm{d}y \\
\mathcal{P}_x &= \int_{R^2} i(u^* u_x - u u_x^*) \,\mathrm{d}x\,\mathrm{d}y \\
\mathcal{P}_y &= \int_{R^2} i(u^* u_y - u u_y^*) \,\mathrm{d}x\,\mathrm{d}y \\
\mathcal{H} &= \int_{R^2} \{\sigma |u_x|^2 + |u_y|^2 + \frac{\kappa}{2}|u|^4 + \frac{\gamma}{2}[(\phi_x)^2 + m_2(\phi_y)^2 \\
&\quad + \lambda(\psi_x)^2 + m_1(\psi_y)^2 + n(\phi_y\psi_x + \phi_x\psi_y)]\} \,\mathrm{d}x\,\mathrm{d}y, \qquad (5)
\end{aligned}$$

corresponding to mass, momentum in the x and y directions and energy, respectively, remain constant in time.

1.2 *Blow-up in the HEE case*

The analytical results reported in the literature for blow-up of solutions in the HEE case can be summarized as follows. Using the pseudo-conformal invariance of the solutions, an explicit blow-up profile

$$\begin{aligned}
u(x, y, t) &= \frac{1}{(a+bt)} \exp\left[ib\frac{y^2 - x^2}{4(a+bt)}\right] \frac{\mu}{1 + A\mu^2(x'^2 + y'^2)} \\
\phi(x, y, t) &= \frac{1}{(a+bt)} \frac{\mu^2 x'}{2[1 + A\mu^2(x'^2 + y'^2)]} \\
\psi(x, y, t) &= \frac{1}{(a+bt)} \frac{\mu^2 y'}{2m_1[1 + A\mu^2(x'^2 + y'^2)]} \qquad (6)
\end{aligned}$$

was obtained in [6] for the HEE case under the following conditions on the physical parameters

$$\sigma = -1, \quad \lambda = m_1(1-n), \quad m_2 = 1 - \frac{n}{m_1}, \quad \kappa = -\frac{\gamma}{2}\left(1 + \frac{1}{m_1}\right). \qquad (7)$$

In Equation (6), μ is an arbitrary real constant, $A = \gamma(1 - (1/m_1))/16$ and the transformed coordinates (x', y', t') are given by

$$x' = \frac{x}{(a+bt)}, \quad y' = \frac{y}{(a+bt)}, \quad t' = \frac{c+dt}{a+bt},$$

where a, b, c, d are real numbers satisfying $ad - bc = 1$.

1.3 *Blow-up in the EEE case*

For the EEE case, the conditions needed for the global existence or global non-existence of the solutions have been discussed in [2,7]. The main results of [2] are as follows:

(A) The solutions of the GDS system will exist globally provided that $\kappa \geq \max\{-\gamma \max(1, 1/m_1), 0\}$ and $m_1 > 0$;
(B) The solutions of the GDS system cannot exist globally when $\kappa < \min(-\gamma/m_1, 0)$ and $m_1 > 1$.

It is to be noted that there remains a gap between the global existence and global nonexistence intervals of κ, where neither a global existence nor a blow-up result is established. Motivated by some estimates of [10] where numerical simulations of the GDS system have been conducted, the above results have been improved in [7]. The improved results of [7] are as follows:

(A) (Theorem 3 of [7]) Suppose that $\lambda > 1$, $m_2 > m_1 > 0$, $n > 0$ and let

$$\delta_1 \equiv \frac{1 + m_1 - 2n}{\lambda m_1 + m_2}.$$

If, in addition to hypothesis in Theorem 3 of [7],

$$\text{(i)} \quad \gamma > 0 \text{ and } \kappa \geq -\gamma \min\left\{\delta_1, \frac{1}{m_1}, 1\right\} \tag{8}$$

or

$$\text{(ii)} \quad \gamma < 0 \text{ and } \kappa \geq -\gamma \max\left(\frac{1}{m_1}, 1\right), \tag{9}$$

then the solutions of the GDS system are global.
(B) (Lemma 2 of [7]) Suppose that $\lambda > 1$, $m_2 > m_1 > 0$, $n > 0$ and let

$$\delta_2 \equiv \frac{\sqrt{m_1}(1 + m_1 - 2n) + \sqrt{\lambda m_2}(1 + m_1)}{(m_1\sqrt{\lambda} + \sqrt{m_1 m_2})(\sqrt{m_2} + \sqrt{\lambda m_1})}.$$

If, in addition to hypothesis in Lemma 2 of [7],

$$\text{(i)} \quad \gamma > 0 \text{ and } \kappa < -\gamma \min\left\{\delta_2, \frac{1}{m_1}, 1\right\} \tag{10}$$

or

$$\text{(ii)} \quad \gamma < 0 \text{ and } \kappa < -\gamma \max\left(\frac{1}{m_1}, 1\right), \tag{11}$$

then the solutions with negative initial energy blow-up in finite time.

It is important to emphasize that the improved results given above remove the gap for negative values of the coupling parameter (i.e. for $\gamma < 0$) and narrow the gap for positive values of the coupling parameter (i.e. for $\gamma > 0$).

2. Split-step Fourier method for the GDS system

In this section, we present the split-step Fourier method for the GDS system. The use of Fourier series dictates periodic boundary conditions in u and ϕ, ψ. Application of the numerical method requires truncation of the infinite xy-plane to a finite rectangular region $[a, b] \times [c, d]$. To simplify the presentation of the Fourier method, the rectangular region $[a, b] \times [c, d]$ is normalized to square region $[0, 2\pi] \times [0, 2\pi]$ using the transformations $X = 2\pi(x-a)/(b-a)$ and $Y = 2\pi(x-c)/(d-c)$ in which the GDS system becomes

$$\begin{aligned} iu_t + \bar{\sigma}u_{XX} + \bar{\alpha}u_{YY} &= \kappa|u|^2u + (\bar{\gamma}_1\phi_X + \bar{\gamma}_2\psi_Y)u \\ \bar{\beta}\phi_{XX} + \bar{m}_2\phi_{YY} + \bar{n}\psi_{XY} &= \bar{\nu}_1(|u|^2)_X \\ \bar{\lambda}\psi_{XX} + \bar{m}_1\psi_{YY} + \bar{n}\phi_{XY} &= \bar{\nu}_2(|u|^2)_Y \end{aligned}$$

where

$$\begin{aligned} &\bar{\sigma} = \left(\frac{2\pi}{b-a}\right)^2\sigma, \quad \bar{\alpha} = \left(\frac{2\pi}{d-c}\right)^2, \quad \bar{\gamma}_1 = \left(\frac{2\pi}{b-a}\right)\gamma, \quad \bar{\gamma}_2 = \left(\frac{2\pi}{d-c}\right)\gamma \\ &\bar{\beta} = \left(\frac{2\pi}{b-a}\right)^2, \quad \bar{m}_2 = \left(\frac{2\pi}{d-c}\right)^2 m_2, \quad \bar{n} = \frac{(2\pi)^2}{(b-a)(d-c)}n \\ &\bar{\lambda} = \left(\frac{2\pi}{b-a}\right)^2\lambda, \quad \bar{m}_1 = \left(\frac{2\pi}{d-c}\right)^2 m_1, \quad \bar{\nu}_1 = \frac{2\pi}{b-a}, \quad \bar{\nu}_2 = \frac{2\pi}{d-c}. \end{aligned}$$

The interval $[0, 2\pi]$ in the X direction is divided into N_X equal subintervals, with grid spacing $\Delta X = 2\pi/N_X$ and the interval $[0, 2\pi]$ in the Y direction is divided into N_Y equal subintervals, with grid spacing $\Delta Y = 2\pi/N_Y$. We assume N_X and N_Y are even, positive integers. The spatial grid points are $(X_j, Y_k) = (j\Delta X, k\Delta Y)$ with $j = 0, 1, 2, \ldots, N_X$ and $k = 0, 1, 2, \ldots, N_Y$. The time interval $[0, T]$ is divided into M equal subintervals with grid spacing $\Delta t = T/M$. The temporal grid points are given by $t_m = m\Delta t, m = 0, 1, 2, \ldots, M$. The approximate values of $u(X_j, Y_k, t_m)$, $\phi(X_j, Y_k, t_m)$ and $\psi(X_j, Y_k, t_m)$ are denoted by $U^m_{j,k}$, $\Phi^m_{j,k}$ and $\Psi^m_{j,k}$, respectively. For the two-dimensional discrete Fourier transform of a function $v(X, Y, t)$ with $V_{j,k}(t) \approx v(X_j, Y_k, t)$ ($j = 0, 1, 2, \ldots, N_X$ and $k = 0, 1, 2, \ldots, N_Y$), the Fourier coefficients are given by

$$\begin{aligned} &\hat{V}_{p,q} = \mathcal{F}_{p,q}[V_{j,k}] = \frac{1}{N_X N_Y}\sum_{j=0}^{N_X-1}\sum_{k=0}^{N_Y-1} V_{j,k}\exp[-i(pX_j + qY_k)], \\ &-\frac{N_X}{2} \le p \le \frac{N_X}{2} - 1, \quad -\frac{N_Y}{2} \le q \le \frac{N_Y}{2} - 1 \end{aligned}$$

where p and q are the corresponding wavenumbers. The inverse discrete Fourier transform is defined as

$$\begin{aligned} &V_{j,k} = \mathcal{F}^{-1}_{j,k}[\hat{V}_{p,q}] = \sum_{p=-N_X/2}^{(N_X/2)-1}\sum_{q=-N_Y/2}^{(N_Y/2)-1} \hat{V}_{p,q}\exp[i(pX_j + qY_k)], \\ &j = 0, 1, 2, \ldots, N_X - 1, \quad k = 0, 1, 2, \ldots, N_Y - 1. \end{aligned}$$

The two-dimensional discrete Fourier transform $\mathcal{F}$ and its inverse $\mathcal{F}^{-1}$ are always computed via a fast Fourier transform (FFT) algorithm.

The initial value problem for the GDS system is decomposed into linear and nonlinear subproblems which involve the linear partial differential equation

$$\mathcal{L}: \; iu_t + \bar{\sigma}u_{XX} + \bar{\alpha}u_{YY} = 0 \tag{12}$$

and the set of nonlinear coupled partial differential equations

$$\mathcal{N}: \quad \bar{\beta}\phi_{XX} + \bar{m}_2\phi_{YY} + \bar{n}\psi_{XY} - \bar{\nu}_1(|u|^2)_X = 0 \tag{13}$$

$$\bar{\lambda}\psi_{XX} + \bar{m}_1\psi_{YY} + \bar{n}\phi_{XY} - \bar{\nu}_2(|u|^2)_Y = 0 \tag{14}$$

$$iu_t - \kappa|u|^2u - (\bar{\gamma}_1\phi_X + \bar{\gamma}_2\psi_Y)u = 0 \tag{15}$$

respectively. The main idea in the split-step method is to approximate the exact solution of the GDS system by solving the linear and nonlinear subproblems in a given sequential order, in which the solution of one subproblem is employed as an initial condition for the next subproblem. We shall use the first-order split-step method in which the linear and the nonlinear steps are taken successively.

In the spectral domain, the linear equation (12) can be rewritten as

$$i\frac{\mathrm{d}\hat{U}_{p,q}}{\mathrm{d}t} - (\bar{\sigma}p^2 + \bar{\alpha}q^2)\hat{U}_{p,q} = 0;$$

that is, spatial differentiations in physical space are converted to multiplications in spectral space. This is a linear, ordinary differential equation for the Fourier coefficient $\hat{U}_{p,q}$ and it can be solved exactly. Therefore, for the linear subproblem, the advancements in time are performed according to

$$U_{j,k}^{m+1} = \mathcal{F}_{j,k}^{-1}\{\exp[-i(\bar{\sigma}p^2 + \bar{\alpha}q^2)\Delta t]\mathcal{F}_{p,q}[U_{j,k}^m]\}. \tag{16}$$

The numerical solution of the linear subproblem requires only two FFT operations.

We now focus on the nonlinear subproblem. We first note that all the nonlinear terms appearing in the nonlinear subproblem are evaluated using a pseudo-spectral approximation. Solving the set of nonlinear coupled partial differential equations given by Equations (13)–(15) involves four steps. The first step is to rewrite Equations (13) and (14) in the spectral domain for a typical time level t_m:

$$-(\bar{\beta}p^2 + \bar{m}_2q^2)\hat{\Phi}_{p,q}^m - \bar{n}pq\hat{\Psi}_{p,q}^m = i\bar{\nu}_1p\hat{\Lambda}_{p,q}^m \tag{17}$$

$$-\bar{n}pq\hat{\Phi}_{p,q}^m - (\bar{\lambda}p^2 + \bar{m}_1q^2)\hat{\Psi}_{p,q}^m = i\bar{\nu}_2q\hat{\Lambda}_{p,q}^m \tag{18}$$

where the Fourier representation

$$\hat{\Lambda}_{p,q}^m = \frac{1}{N_XN_Y}\sum_{j=0}^{N_X-1}\sum_{k=0}^{N_Y-1}|U_{j,k}^m|^2\exp[-i(pX_j + qY_k)],$$

$$-\frac{N_X}{2} \le p \le \frac{N_X}{2} - 1, \qquad -\frac{N_Y}{2} \le q \le \frac{N_Y}{2} - 1$$

is used. The second step is to solve the system of two simultaneous linear equation, Equations (17) and (18), for the Fourier coefficients $\hat{\Phi}_{p,q}^m$ and $\hat{\Psi}_{p,q}^m$:

$$\hat{\Phi}_{p,q}^m = \frac{i[(\bar{\nu}_2\bar{n} - \bar{\nu}_1\bar{m}_1)q^2p - \bar{\lambda}\bar{\nu}_1p^3]\hat{\Lambda}_{p,q}^m}{[(\bar{\lambda}p^2 + \bar{m}_1q^2)(\bar{\beta}p^2 + \bar{m}_2q^2) - \bar{n}^2p^2q^2]} \tag{19}$$

$$\hat{\Psi}_{p,q}^m = \frac{i[(\bar{\nu}_1\bar{n} - \bar{\nu}_2\bar{\beta})p^2q - \bar{m}_2\bar{\nu}_2q^3]\hat{\Lambda}_{p,q}^m}{[(\bar{\lambda}p^2 + \bar{m}_1q^2)(\bar{\beta}p^2 + \bar{m}_2q^2) - \bar{n}^2p^2q^2]}. \tag{20}$$

The third step is to compute the values of $(\phi_X)^m_{j,k}$ and $(\psi_Y)^m_{j,k}$ using Equations (19) and (20). To obtain spectral representations of the spatial derivatives ϕ_X and ψ_Y, Equations (19) and (20) are multiplied by ip and iq, respectively. Then physical space representations of ϕ_X and ψ_Y are obtained by employing the inverse Fourier transform:

$$(\phi_X)^m_{j,k} = \mathcal{F}^{-1}_{j,k}\left\{\frac{[(\bar{\nu}_1\bar{m}_1 - \bar{\nu}_2\bar{n})p^2q^2 + \bar{\lambda}\bar{\nu}_1 p^4]\hat{\Lambda}^m_{p,q}}{[(\bar{\lambda}p^2 + \bar{m}_1q^2)(\bar{\beta}p^2 + \bar{m}_2q^2) - \bar{n}^2p^2q^2]}\right\} \tag{21}$$

$$(\psi_Y)^m_{j,k} = \mathcal{F}^{-1}_{j,k}\left\{\frac{[(\bar{\nu}_2\bar{\beta} - \bar{\nu}_1\bar{n})p^2q^2 + \bar{m}_2\bar{\nu}_2 q^4]\hat{\Lambda}^m_{p,q}}{[(\bar{\lambda}p^2 + \bar{m}_1q^2)(\bar{\beta}p^2 + \bar{m}_2q^2) - \bar{n}^2p^2q^2]}\right\}. \tag{22}$$

The final step is to solve the nonlinear equation (15). For this aim, we substitute $u = r\exp(i\theta)$ into Equation (15), where r and θ are assumed to be real functions of X, Y and t. The resulting equations for r and θ are

$$r_t = 0, \quad \theta_t = -(\kappa r^2 + \bar{\gamma}_1\phi_X + \bar{\gamma}_2\psi_Y).$$

The first equation implies that r is independent of time: $r = R(X, Y)$. By substituting $|u|^2 = R^2(X, Y)$ into Equations (13) and (14), we can get that ϕ and ψ are time-independent quantities. So the right-hand side of the second equation does not depend on time and a solution is $\theta = -(\kappa R^2 + \bar{\gamma}_1\phi_X + \bar{\gamma}_2\psi_Y)t + \theta_0$. We have the explicit solution of the nonlinear subproblem in the form

$$u(X, Y, t_{m+1}) = u(X, Y, t_m)\exp[-i(\kappa|u(X, Y, t_m)|^2 + \bar{\gamma}_1\phi_X(X, Y, t_m) + \bar{\gamma}_2\psi_Y(X, Y, t_m))\Delta t].$$

Finally, the advancements in time are performed according to

$$U^{m+1}_{j,k} = \exp\{-i[\kappa|U^m_{j,k}|^2 + \bar{\gamma}_1(\phi_X)^m_{j,k} + \bar{\gamma}_2(\psi_Y)^m_{j,k}]\Delta t\}U^m_{j,k} \tag{23}$$

for the nonlinear subproblem.

A brief summary of the first-order split-step Fourier method can be given as follows. Given the data $U^m_{j,k}$ at any time $t = t_m$, first compute the values of $(\phi_X)^m_{j,k}$ and $(\psi_Y)^m_{j,k}$ from Equations (21) and (22), respectively. Then, advance the solution according to Equation (23) to produce an intermediate solution. Use this intermediate solution as the initial data for the linear subproblem. That is, advance the solution according to Equation (16) to produce the solution at time $t = t_{m+1}$. This completes the one time step description of the first-order split-step method. Repeated applications of the operations yield an explicit numerical scheme which is of spectral-order accuracy in space and first-order accuracy in time.

3. Numerical experiments

We now test the ability of the split-step Fourier method to detect blow-up solutions of the GDS system by comparing the numerical results with the analytical results available in the literature.

3.1 *HEE case*

In the HEE case ($\sigma = -1$), we numerically test the split-step Fourier method by using the analytical blow-up profile given in Equations (6) and (7). Our purpose is to show that the split-step Fourier method does not miss the blow-up phenomena and that the numerical blow-up time approximates the analytical blow-up time. If we choose $a = 1$, $b = -4$, $\mu = 1$ and $A = 1$, the analytic blow-up profile given in Equation (6) takes the following form:

$$u(x, y, t) = \frac{1}{(1-4t)} \exp\left[i \frac{x^2 - y^2}{(1-4t)}\right] \frac{1}{1 + (x^2 + y^2)/(1-4t)^2}. \quad (24)$$

The L_∞ norm of the exact solution (24) is $\|u\|_\infty = 1/(1-4t)$ from which we deduce that the analytical blow-up time is $t_* = 0.25$. The initial condition corresponding to the exact solution (24) is given by

$$u(x, y, 0) = \exp[i(x^2 - y^2)] \frac{1}{1 + x^2 + y^2}. \quad (25)$$

The values of physical parameters, used in the present computations, are $m_1 = 2$, $m_2 = n = \lambda = 2/3$, $\gamma = 32$, $\kappa = -24$ and they are compatible with Equation (7).

The problem is solved on the square region $[-40, 40] \times [-40, 40]$ for times up to $T = 0.25$. We choose the value of Δt for a given Δx $(\equiv (b-a)/N_X)$ and Δy $(\equiv (d-c)/N_Y)$ so that $\Delta t = \nu[(\Delta x)^2 + (\Delta y)^2]$, where the value of ν is fixed at $\nu = 0.1$. We present both the numerical blow-up time and the corresponding amplitude of the approximate solution for various numbers of spatial and temporal grid points in Table 1. As we increase the number of grid points in both space and time, we observe that the numerical blow-up time approximates the analytical blow-up time. Moreover, the corresponding amplitude of the solution increases with the number of grid points. These results show that the split-step Fourier method does not miss the blow-up phenomena and the prediction for the blow-up time seems satisfactory.

We present the variation of the L_∞ norm of the approximate solution with time in Figure 1. We point out that the initial amplitude is one in the initial condition given by Equation (25). However, the amplitude of the numerical solution increases as time increases and it becomes approximately 19 near the numerical blow-up time. The profile of the numerical solution near the blow-up time

Table 1. Numerical blow-up time and the corresponding amplitude, $\|u\|_\infty$, of solution ($\nu = 0.1$).

$N_X = N_Y$	M	t_*	$\|u\|_\infty$
256	13	0.23076923	5.238
512	51	0.24509804	9.840
1024	205	0.24878049	18.834

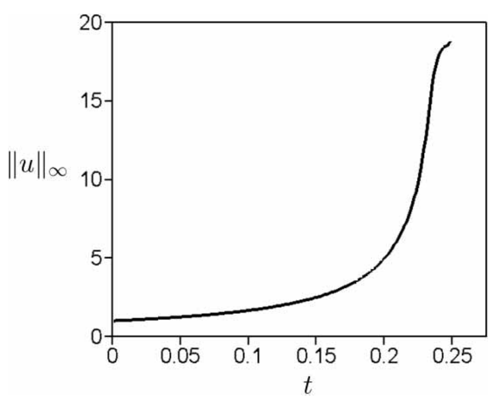

Figure 1. The variation of $\|u\|_\infty$ with time.

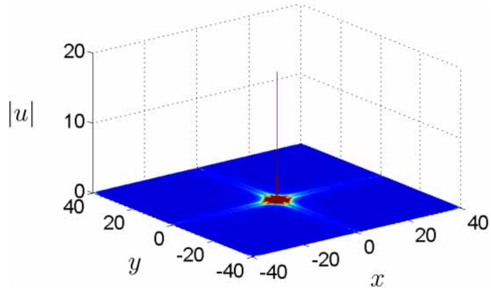

Figure 2. The blow-up profile near the blow-up time ($t = 0.24878049$).

is presented in Figure 2. We conclude that, for the HEE case, the split-step Fourier method is very successful to detect blow-up solutions of the GDS system and to compute the corresponding blow-up time.

3.2 *EEE case*

In the EEE case ($\sigma = 1$), we compare the numerical results with the analytical results given in Equations (8)–(11). We consider the Gaussian initial condition

$$u(x, y, 0) = 6 \exp\left(-\frac{x^2 + y^2}{4}\right). \tag{26}$$

For all computations presented here, the problem is solved on the square region $[-12, 12] \times [-12, 12]$ for times up to $T = 1$. The values of physical parameters are $m_1 = 2$, $m_2 = 3$, $n = 1$, $\lambda = 2$ which are compatible with Equation (4). Below, we consider separately the cases corresponding to negative and positive values of the coupling parameter γ.

First, we consider the case $\gamma < 0$. For convenience, we set $\gamma = -1$. For the chosen values of physical parameters, it follows from Equations (8)–(11) that the solution exists globally when $\kappa \geq 1$ and that the solution blows-up when $\kappa < 1$. So there is no gap interval when $\gamma < 0$. In Figure 3, the variation of $\|u\|_\infty$ with time for the values of $\kappa = 1$ and $\kappa = 2$ is presented. We observe that, in both cases, the amplitude of numerical solution decreases as time increases. This is a strong indication of absence of blow-up when $\kappa = 1$ or $\kappa = 2$. Similarly, in Figure 4 the variation of $\|u\|_\infty$ with time for the values of $\kappa = 0$ and $\kappa = -1$ is presented. For both values of κ, the amplitude of numerical solution increases as time increases. This is a strong indication

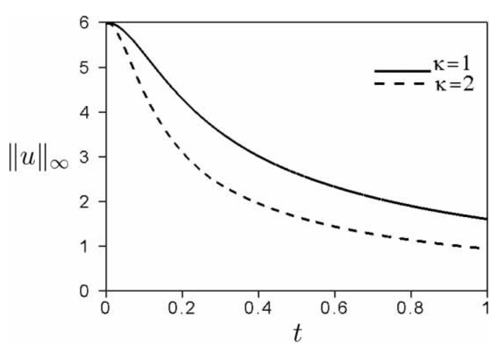

Figure 3. The variation of $\|u\|_\infty$ with time for the values of $\kappa = 1$ and $\kappa = 2$.

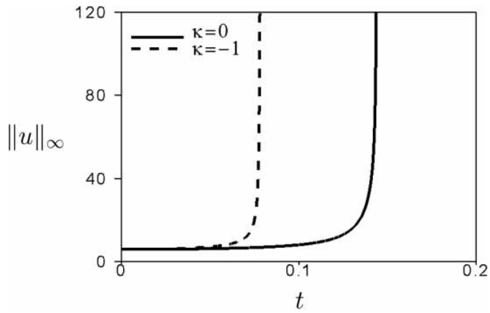

Figure 4. The variation of $\|u\|_\infty$ with time for the values of $\kappa = 0$ and $\kappa = -1$.

of that the solutions blow-up in finite time. So the numerical results obtained for both global existence and blow-up are in good agreement with the analytical results when $\gamma < 0$.

Second, we consider the case $\gamma > 0$. For convenience, we set $\gamma = 1$. For the chosen values of physical parameters, it follows from Equations (8)–(11) that the solution exists globally when $\kappa \geq -1/7$ and that the solution blows-up when $\kappa < (29 - 17\sqrt{3})$. It must be noted that there is no analytical results in the literature for the interval $(29 - 17\sqrt{3}) \leq \kappa < -1/7$. In other words, for this gap interval, neither a global existence nor a blow-up result is established. We now test the suggested split-step Fourier method with available analytical results. We first present the variation of $\|u\|_\infty$ with time for the values of $\kappa = -1/7$ and $\kappa = 0$ in Figure 5. In both cases, the amplitude

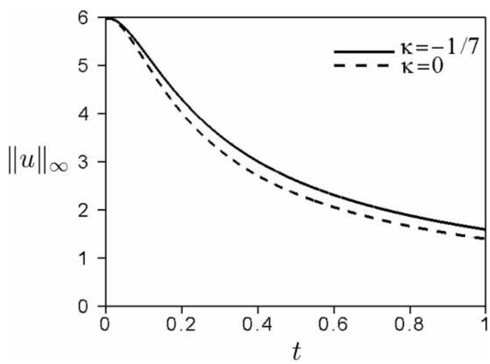

Figure 5. The variation of $\|u\|_\infty$ with time for the values of $\kappa = -1/7$ and $\kappa = 0$.

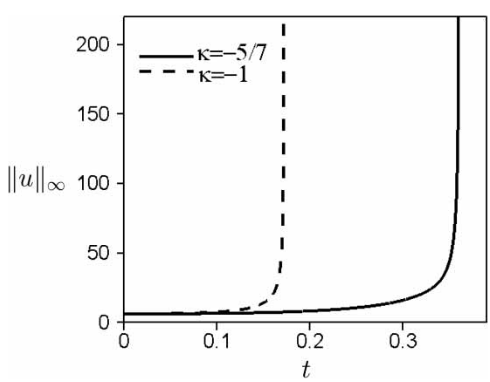

Figure 6. The variation of $\|u\|_\infty$ with time for the values of $\kappa = -5/7$ and $\kappa = -1$.

of approximate solution decreases as time increases, that is, the solution does not blow-up. In Figure 6, the variation of $\|u\|_\infty$ with time for the values of $\kappa = -5/7$ and $\kappa = -1$ is presented. We observe that in both cases, the amplitude of solution increases as time increases, which is an indication of blow-up. So the numerical results obtained for both global existence and blow-up are in good agreement with the analytical results available in the literature. We do not provide any numerical results corresponding to the gap interval $(29 - 17\sqrt{3}) \leq \kappa < -1/7$. The reason is that the initial energy is positive for any value of κ in the gap interval.

4. Conclusions

A split-step Fourier method is presented to determine whether a singularity develops in the solution of the GDS system at a finite time. The numerical method takes advantage of the spectral accuracy of Fourier series in space and it employs a first-order time-splitting approach. The strength of the method lies in the fact that the nonlinear subproblem possesses a simple analytical solution. Numerical experiments on problems from the literature are performed to measure how capable the method is in capturing the blow-up phenomena. The numerical tests suggest that the present scheme is accurate and reliable to detect the presence of a singularity. An important advantage of the method is the ability to detect the blow-up phenomena on a reasonable coarse mesh. It seems to have some advantage over standard finite difference methods in capturing the blow-up phenomena. It remains to be seen what advantages split-step Fourier methods offer in other nonlinear partial differential equations where singularities develop.

References

[1] C. Babaoglu and S. Erbay, *Two dimensional wave packets in an elastic solid with couple stresses*, Int. J. Nonlinear Mech. 39 (2004), pp. 941–949.
[2] C. Babaoglu, A. Eden, and S. Erbay, *Global existence and nonexistence results for a generalized Davey–Stewartson system*, J. Phys. A, Math. Gen. 37 (2004), pp. 11531–11546.
[3] C. Besse , N.J. Mauser, and H.P. Stimming, *Numerical study of the Davey-Stewartson system*, M2AN Math. Model. Numer. Anal. 38 (2004), pp. 1035–1054.
[4] H. Borluk, G.M. Muslu, and H.A. Erbay, *A numerical study of the long wave-short wave interaction equations*, Math. Comput. Simulation 74 (2007), pp. 113–125.
[5] A. Davey and K. Stewartson, *On three-dimensional packets of surface wawes*, Proc. R. Soc. Lond. A 338 (1974), pp. 101–110.
[6] A. Eden, H.A. Erbay, and G.M. Muslu, *Two remarks on a generalized Davey–Stewartson system*, Nonlinear Anal. TMA 64 (2006), pp. 979–986.
[7] A. Eden, H.A. Erbay, and G.M. Muslu, *Closing the gap in the purely elliptic generalized Davey–Stewartson system*, Nonlinear Anal. TMA 69 (2008), pp. 2575–2581.
[8] M. McConnell, A.S. Fokas, and B. Pelloni, *Localised coherent solutions of the DSI and DSII equations – a numerical study*, Math. Comput. Simulation 69 (2005), pp. 424–438.
[9] R.I. McLachlan and G.R.W. Quispel, *Splitting methods*, Acta Numer. 11 (2002), pp. 341–434.
[10] G.M. Muslu, *Blow-up solutions of the generalized Davey–Stewartson system: Analytical and numerical results*, Ph.D. thesis, Department of Mathematics, Istanbul Technical University, 2007.
[11] G.M. Muslu and H.A. Erbay, *A split-step Fourier method for the complex modified Korteweg-de Vries equation*, Comput. Math. Appl. 45 (2003), pp. 503–514.
[12] G.M. Muslu and H.A. Erbay, *Higher-order split-step Fourier schemes for the generalized nonlinear Schrödinger equation*, Math. Comput. Simulation 67 (2005), pp. 581–595.
[13] T.R. Taha and M.J. Ablowitz, *Analytical and numerical aspects of certain nonlinear evolution equations. II. Numerical, nonlinear Schrödinger equation*, J. Comput. Phys. 55 (1984), pp. 203–230.
[14] P.W. White and J.A.C. Weideman, *Numerical simulation of solitons and drominions in the Davey–Stewartson system*, Math. Comput. Simulation 37 (1994), pp. 469–479.