Short-term Price Prediction in Electricity Market Using Nonlinear Excess Demand Specification

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Abstract

Forecasting short term price movements in the electricity market is not a trivial task. Variety of models and ideas have been proposed for electricity price forecasting in the literature. We present a dynamic model of Turkish hourly electricity prices for 2010 to 2016 periods. A stochastic model of excess demand is proposed to describe the price formation dynamics and to predict short term price behavior. A maximum likelihood estimator is derived via a nonlinear reaction function of prices to the latent factor excess demand. The excess demand is defined as a normally distributed random variable conditional on variables derived from observable prices and volumes from the previous days. The conditional mean and variance of the price process are derived to compare the model implied predictions to a linear alternative for illustration. The aim of the study is not full forecast comparison of alternating models, but rather to introduce the

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nonlinear excess demand specification as a forecasting tool. The excess demand model performs reasonably well in the next day price predictions for 2010-2016 periods. Also the nonlinear structure has time varying volatility which along with the nonlinear mean function, brings two important features of time series modelling dynamics together in parsimonious model.

Keywords: Electricity, Energy forecasting, Excess Demand, Nonlinear Modelling, Nonlinear time series, Prediction intervals. **JEL classification**: C01, C13, C51, C53, Q47.

1 Introduction

Over the last few decades, electricity markets has been liberalized and deregulated in order to provide long run competition and efficiency gains. Electricity trade is no different from other commodity trades and considered as a technical oriented business (Murthy, Sedidi and Panda, 2014). However, electricity trade has some unique characteristics that differs it from other commodity markets. These characteristics include; electricity can not be stored, supply and demand should be met simultaneously and there is strong seasonality in end user demand. Forecasting electricity prices has long term, medium term and short term objectives. Long term objective of price forecasting focus on investment profitability and planning analysis, medium term forecasting objective focus on risk management, balance sheet analysis and derivative pricing analyses. Short term objective of electricity price forecasting focus hedging from volatility of the market and the prices (Misiorek, Trueck and Weron, 2006). Volatile nature of electricity market which depends on real time balancing of electrical supply and demand leads problems especially for price forecasting. The factors that affect the supply and demand balance also play an important role in price volatility also they affect the spot electricity prices. Such factors include, power station interruption, imperfect transmission grid reliability, weather conditions changes, related commodity price changes including fuel prices. (Girish, 2012, Weron, 2006). Also with the increasing number of participants in the electricity market as suppliers and distributors with differential roles in the supply-chain, strategic interactions can also be an important factor. This makes spot electricity price forecasting critical for all participants of electricity market.

In this paper, we present a dynamic forecasting model for hourly prices of electricity in Turkey. The model addresses the challenge of forecasting in a market where the structure has been changed dramatically. The electricity market transformed in Turkey to adapt a competitive market from a oligopolist state operated structure, therefore pricing is based on demand only recently. The recent change in the market structure brings modern statistical techniques to bear on analysis in a forecasting environment long dominated by predictions based on the immediate experience of industry practitioners, or prediction tools only concerned about the supply side of the market. Our approach is based on a nonlinear price adjustment equation modelling where daily electricity prices change in response to unobserved excess demand which is itself modeled as a stochastic linear function of observable variables in the electricity market. This estimation strategy helps us to capture (i) the nonlinearity in the reaction of the price changes to the excess demand, i.e. we observe when price changes are small, a linear relationship of the price change with the excess demand adequately presents the market behavior, however for large price changes this is no longer the case; (ii) the process of the demand more realistically, i.e.: the excess demand function depends on an unobserved component as well as the observables, therefore the model is flexible to absorb sudden price changes and also it implies a nonconstant variance for the price process.

Short term spot electricity price forecasting techniques have emerged by contributions from several literatures. Among those, employing statistical process modeling by electrical engineering and econometrics techniques by financial economics literatures opened the venue for a vast amount of research to be conducted on price prediction and they contributed substantially. Short term spot electricity price forecasting techniques broadly include game theory models, simulation models and the time-series models (Aggarwal, Mohan and Kumar, 2009). Time series models can be classified as regression based models and Artificial Neural Networks based models. Regression based models mostly employ both linear and non-linear models and compare their forecast performance in order to find the best fit model. Regression based models employed in spot electricity price forecasting includes, AR/ARMA models which explain the data with itself without any explanatory variables, ARCH type linear or non-linear conditional heteroscadasticity models, non-linear Markov Switching type models and stochastic models.

Contreras et al. (2003) for Spain and Californian energy markets , Cauresma et al. (2004) for Leipzig Power Exchange (LPX) Germany, Zhou et al. (2004) for California power markets, Kristiansen (2012) for Nord Pool power market employed ARIMA type models for spot electricity price forecasting. They estimated alternative ARIMA models and compare their forecast performance according to some selected forecast performance criteria. Weron and Misiorek (2005) employed ARMA and ARMAX Model (ARMA model with exogenous variable) for California power markets spot electricity price forecasting and found that ARX and ARMAX models superior from ARIMA model. The papers employ ARCH type models for spot electricity price forecasting include, Garcia et al. (2005), Bowden and Payne (2008), Gianfreda & Grossi (2012) and Hickey et al. (2012). Garcia et al. (2005) employ 2 alternative GARCH model to predict spot electricity prices for Spain and California power markets. They found that average errors in the Spanish market and California Market are found %7 and %4 respectively. Bowden and Payne (2008) estimates ARIMA, ARIMA-EGARCH, and ARIMA-EGARCH-M models in order to estimate spot electricity prices for each 5 hubs of Midwest Independent System Operator (MISO) and compare the alternative models according to their in sample and out-of sample forecast performance. They found that according to in sample forecasting performance any of the models dominates the others. However, ARIMA-EGARCH-Model has been found superior according to out-of sample performance except Michigan hub. Gianfreda & Grossi (2012) investigated how technologies, market concentration, congestions and volumes variables affect the zonal prices for Italian Electricity Zonal Market by employing Implementing Reg-ARFIMA–GARCH models and found that the forecast performance of the selected model increase when they considered these factors. Hickey et al. (2012) employs

4 alternative ARMAX-GARCH model in order to forecast spot electricity price of 5 MISO hubs and compare the alternative models according to their forecast performance. They found that APARCH models forecast performance has been superior in deregulated states and volatility characteristics of the regulated states could be captured by simple GARCH model. The papers employ Markov Switching type models for spot electricity price forecasting include, Weron et al. (2004), Weron and Misiorek (2006), Weron and Misiorek (2008). Weron et al. (2004) tried to forecast spot electricity prices for Nordic Power Exchange by employing different Regime Switching models which exhibit mean reversion and jump behavior. They found high similarity with real price data; however, they found high price spike and extreme event form real data also. Weron and Misiorek (2006) investigate alternative time series models including non-linear regime switching TAR-type models for spot electricity prices for California Power market. They found that the non linear models performance including regime switching TAR models was disappointing and they support adequacy of linear models for forecasting. Weron and Misiorek (2008) employ 12 alternative time series model for spot electricity price forecasting including threshold and Markov models for California and Nordic markets and found that models which take into consideration of the system load has better performance form pure price models and semiparametric models generally gives better point and interval forecasts than their competitors.

Lastly there are papers which employ ANN (Artificial Neural Network) models for spot electricity price forecasting as Pao (2007) for Leipzig Power Market and Catalao at. all (2007) for California and Spain Power Markets. They compare the forecast performance of non-linear ANN model with alternative ARIMA models and found that ANN models forecast performance was superior from ARIMA type models. Ghodsi and Zakerinia (2012) tried to forecast spot electricity prices for Ontario electricity market by employing ARIMA models, ANN models and fuzzy regression model and found that fuzzy regression model's forecast performance is the best. Strengths and weakness, degree of complexity of the models employed by various approaches in the electricity price forecasting literature are addressed in a recent survey article by Weron (2014).

In sum, the models which take into consideration the non-linearity in the modelling has been found superior to linear models in the literature except maybe the work by Weron and Misiorek (2006). Our study also fits into the nonlinear price modelling literature. However our approach differs itself from pure statistical techniques, by exploring the economics behind the price movements. Excess demand models in price prediction considered before by Hendry et al. (1982) and Richard and Zhang (1996) for the UK housing market. More recently Giarratani, Richard and Soytas (2015) applied the excess demand framework for the US scrap steel prices. The latent excess demand structure is proved to be a useful tool to understand the price dynamics in a volatile market environment. In this paper, similar to Giarratani et al. (2015), a latent demand function is estimated and its movements along with the price prediction is demonstrated. The daily prices for the electricity market in Turkey is determined by matching the sell and buy orders of the previous day; hence subsequently by fitting an equilibrium price based on supply and demand curves for the electricity market (proposed for the next day). From an economic point of view, the prices in this scheme can only change due to two main sources: (1) changes in the seller's cost, (2) changes in the buyer's willingness to pay. The price changes in the short run though likely to be affected by the demand related changes more often. Assuming the supply is inelastic in the short run, and controlling for the effects of the economic fundamentals that can affect the supply curve, the remaining variation in prices can be attributed to the change in the electricity demand. With this argument, the short run price fluctuations can be approximated by the changes in the demand curve. Therefore we will introduce a latent factor which we will name as excess demand. Excess demand will be positive whenever the price movement is upward, and it will be negative when the price movement is downward. Therefore implicit in this assumption, we expect the prices not to change if the excess demand is zero.

The relationship between the excess demand and the price changes is positive. Therefore we can predict the direction of the price movement by observing the excess demand. However the reaction of price in general will depend on the price elasticity of demand (supply) which often is not constant over the demand (supply) curve. A linear model of price changes will assume this elasticity is constant. In other words, independent of the change in the excess demand from one day to the other, the reaction of the pice will be proportional to this change. This might be a good approximation when the change in the demand (supply) is relatively small from one day to the other, however likely to perform poor whenever the excess demand increases or decreases significantly over a day. In this case constant elasticity assumption is very much likely to be violated. In the paper, a nonlinear excess demand model is estimated and compared to a linear alternative estimated by OLS. We also employed other time series regression models frequently used in the empirical literature including AR/ARMA models, ARDL model and Markov Switching model in order to compare forecasting performance of the alternative models as robustness check along with a base comparison model of OLS.

The rest of the paper is organized as follows. Section 2 briefly discusses the market structure and price formation in the Turkish electricity market. Section 3 introduces the excess demand framework, develops the nonlinear excess demand specification, and discusses the estimation steps. Section 4 introduces the data and the variables used in the estimation of daily price changes. Section 5 presents the estimation results and compares the performance of the nonlinear model against a linear alternative. In Section 6 forecasting performance of the excess demand model is demonstrated. Finally section 7 concludes. The details and proofs of some derivations are given in the appendix of the paper.

2 Market Structure

Table 1 presents the information available from the market in a particular day. All the data are handled and formed by the Energy Exchange Istanbul (EXIST). The price dynamics during a day is partially determined by a matching mechanism coming from a Day-Ahead market[1]. In this structure the market participants quote their bid and ask prices to the market maker (EXIST) and, a unique price is determined for each hour for the following day. The hour, day, month, and year information of transaction volume and the unique hourly prices are available for research purposes. There are 24 prices from 12:00 A.M. to 11:00 P.M. in 24 hourly increments.

Table 1: Variables

	Variable	Explanation
1	weekday	day of the week
2	hour	hour of the day
3	day	day information
4	month	month information
5	year	year information
6	price	price level
7	volume	volume level

In Figure 1, the daily prices determined from this market structure is shown for a particular day. We see immediately from Figure 1 that the price levels for different hours can vary substantially during the day. This is a general characteristics of a market such as electricity where the production at different hours can be affected by many natural factors. The lack of an efficient storage technology also is a major characteristics of this market which otherwise would be used to smooth the prices during the day.

Every day until 11:30, market participants who participate in the day-ahead market notify the Market Operator of their day-ahead market offers for the following day via the MMS system[2]. A warrant/collateral check is performed before the opening of the market between 11:30 A.M.-12:00 P.M. in order to determine whether the offer qualifies to the Day-Ahead Market. Every day-ahead market offer that has been notified is

[1] There are two price series in a particular day. One is the price determined in the Day-Ahead Market which is the series taken into consideration in this paper. The other is the spot price during the day which can change in response to unforeseen hourly mismatches during the day in the proposed electricty demand/suppy from the previous day.

[2] The details of the market making process can be found at the Energy Exchange İstanbul (EXIST) website at www.exist.com.

assessed and verified by the Market Operator between 11:30 A.M.-12:00 P.M. Each offer that has been verified is assessed via the optimization tool between 12:00 A.M.-01:00 P.M. and market exchange (barter) prices and market exchange (barter) amounts are determined for each hour of the day considered. Related market participants are notified regarding the commercial transaction approvals containing the amounts of approved purchases and sales at 01:00 P.M. every day. In case errors are encountered in these notifications, market participants can make their objections between 01:00 P.M.-01:30 P.M. These objections are assessed between 01:00 P.M.-01:30 P.M. and the participants are notified regarding the results of the assessment. At 02:00 P.M., the prices and match-ups for the 24 hours of the following day are conclusively notified to the participants. Between 00:00 A.M.-04:00 P.M. every day, notifications regarding bilateral agreements by market participants are recorded in the MMS system. The match-up transaction is conducted by the generation of the supply-demand curves through the assessment of 24 hour offers via the optimization tool. In the match-up procedure, first, the supply and demand curve is generated taking into consideration the hourly offers made by the participants and the market exchange (barter) amount and price is determined for each trade range, at the point where the supply-demand curve intersect. Then, block offers are evaluated in turn via the optimization tool and block offers which reduce the cost of the transactions are taken into consideration. Supply-demand curves are formed with the inclusion of the block offers. Finally, elastic offers are taken into consideration and final market exchange (barter) prices and amounts are determined for each hour.

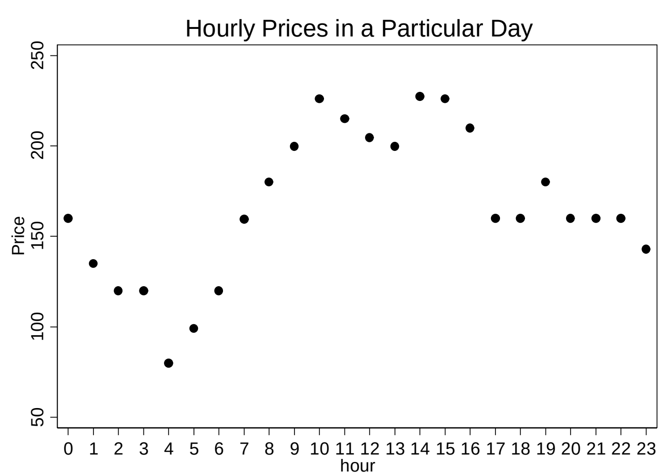

Figure 1: Hourly Prices

One of the innovations that was brought about in the new era kicked off by the emergence of the Day-Ahead Market is that the demand side can now determine the amount that it will be liable to in accordance with the price level[3]. This way, the demand side now has the opportunity to protect itself against the price that will be formed while taking a more active stance in the market. One other innovation is that, the Day-Ahead Market is portfolio-based and every participant can balance its own portfolio. This way, the road has been paved for the participants to present a more balanced structure to the market and reduce the amount of imbalance of the units in their portfolios, though participation is not compulsory in the Day-Ahead Market. Another innovation is that the Day-Ahead Market is reconciled daily and the reconciliation of the debts and receivables of the participants emanating from the commercial transactions is performed the day following the trade. This way, market participants are able to take the payment for the electricity that they have produced on a daily basis rather than at the end of the month, and can thus continue with their investments without cash problems. Finally the last innovation to mention is the warrant mechanism in the Day-Ahead Market. The goal of the warrant mechanism is to secure electricity market and market participants and therefore minimize the negative effects of possible cash problems on the market.

3 Model

3.1 Linear Model

The linear model with the excess demand structure can be described as follows. Let the log differenced price Δp_t be defined as:

$$\Delta p_t = g(E_t;\phi) \tag{1}$$

where $g(E_t;\phi)$ is a linear reaction function of the excess demand E_t for the electricity market and let E_t be defined as a linear function of the observable explanatory variables Z_t and the unobserved variable v_t that affect the excess demand in the electricity market. The coefficient vector γ collects the impact of each variable in Z_t on the excess demand.

$$g(E_t;\phi) = E_t, \quad E_t = \gamma' Z_t + v_t \tag{2}$$

This specification implies that the differenced price Δp_t can be written as a function of the variables that determine the excess demand. We can put the definition for the function $g(E_t;\phi)$ in equation (1) using equation (2) for the excess demand.

[3] See www.exist.com for more information.

$$\Delta p_t = \gamma' Z_t + v_t \tag{3}$$

The specification given in equation (3) has been used as a benchmark for modelling price change in the electricity as well as many other commodity markets. The linear specification and the explanatory variables in the vector Z_t are chosen either depending on theory or as it is done most often in the literature, they are chosen based on theoretical considerations which provide the necessary guideline to the selection of variables, but the linear specification is used as an approximation to the true process. However linear modelling can be short of capturing the true dynamics in the price changes. This might especially be an issue for the electricity market where the demand response to prices can be sensitive to the magnitude of the price change. For this purpose we propose a more flexible reaction function $g(E_t; \phi)$ to capture the nonlinear price change effects of the excess demand. This specification though comes with a cost in terms of estimation of the model. The linear model in equation (3) can be easily estimated using regression based methods by paying attention to the time series dynamics through error-correction frameworks and/or ARMA modelling. However a different specification of the function $g(E_t; \phi)$ than a linear one requires nontrivial derivation of the estimation equations which eventually requires a maximum likelihood estimation.

3.2 Linear Model Extensions

Obviously the specification given in equations (1), (2) and (3) can be easily generalized to account for AR and ARMA specifications. The flexible structure presented in equation (3) in the case of AR or ARMA will include lag values of the log differenced price Δp_t and/or v_t in the vector Z_t.

3.3 Nonlinear Excess Demand Model

The dynamics of the nonlinear model differentiates itself from the linear model in terms of the functional form of the reaction function. Log differenced price Δp_t can still be defined:

$$\Delta p_t = g(E_t; \phi) \tag{4}$$

whereas the function $g(E_t; \phi)$ is a now a nonlinear function of the excess demand E_t for the electricity market[4]:

[4]A cubic form is choosen for the excess demand as in Richard and Zang (1996), and Giarratani, Richard and Soytas (2015). Obviously any other nonlinear function can also be applied. Cubic function has the attractiveness since the cubic term captures the disproportionate response of the

$$g(E_t;\phi) = E_t + \phi_2 E_t^2 + \phi_3 E_t^3 \tag{5}$$

$$E_t = \gamma' Z_t + v_t \tag{6}$$

where the coefficients in the nonlinear reaction function are required to satisfy the following restrictions:

$$\phi_2 > 0, \phi_3 > 0, \phi_3 > \frac{\phi_2^2}{3} \tag{7}$$

The constraints are set in order to ensure that the increase in excess demand is associated with a positive price movement. This is $\frac{\partial g(E_t;\phi)}{\partial E_t} > 0$. To obtain an estimation equation from this specification is not a trivial task since replacing the nonlinear reaction function for the excess demand from equation (5) into equation (4) is no longer an option. The equation that would be obtained this way will include higher order terms in the unobserved component of the excess demand v_t, and the interaction terms that include both v_t and the observables Z_t. Therefore the linear model in equation (3) can only be obtained and subsequently can be estimated using regression based methods under the linear reaction function. We will describe the procedure that we will follow to estimate the parameters of the above model in the nonlinear case. There are several steps to obtain the likelihood function in this model and it requires derivation of the conditional distribution of the log differenced price Δp_t.

3.4 Derivations

The estimation of the nonlinear models as the one we presented above generally starts with initial selection of regressors to be used in the estimation. For instance Hendry (1984) uses a similar modelling of the excess demand to estimate the housing demand in UK, but determined the set of regressors to be used from a extensive specification test in an earlier step. The main difficulty for the nonlinear models is the cost of applying a recursive framework to eliminate the set of regressors. This will require multiple solutions to the model under the given constraints and requires in general a numeric solution to the objective function (for instance: the likelihood function). In this paper we focus on the forecasting performance of the nonlinear excess demand specification, so we rely on the variables that prove to be useful in predicting the differenced price. A specification test using the model is a topic of another paper.

price to large changes in excess demand. Also the quadratic term gives a natural free parameter to test the asymmetric response to positive and negative excess demand.

3.4.1 Inversion of the Price Equation I

We propose the following inversion of equation (4)

$$E_t = h(\Delta p_t; \phi) \tag{8}$$

Substituting this into (6)

$$h(\Delta p_t; \phi) = \gamma' Z_t + v_t \tag{9}$$

Equation (9) is fairly straightforward least squares (LS) problem. It is linear in γ given ϕ, and nonlinear in ϕ. Combining analytical and numerical methods, we can find Maximum Likelihood (ML) estimates of (γ, ϕ).

3.4.2 Reparametrization of the Excess Demand

The first step consists of reparametrization of equation (4) in a way which simplifies the inversion in (8). Note that under (5), the cubic equation has a unique real solution in E_t. One can employ a solution to the cubic equation in a number of different ways. We proceed with Cardano's method. However the key is to introduce a linear transformation to E_t which sets the second order coefficient of the cubic equal to 0. Let

$$E_t = \phi_3^{-\frac{1}{3}} y_t - \frac{\phi_2}{3\phi_3} \tag{10}$$

Then equation (5) becomes

$$\Delta p_t = y_t^3 + a y_t + b \tag{11}$$

and the coefficients of the new transformation is related to the coefficients ϕ as follows:

$$\begin{aligned} a &= \phi_3^{-\frac{1}{3}} \left(1 - \frac{\phi_2^2}{3\phi_3}\right) > 0 \\ b &= \frac{1}{3}\frac{\phi_2}{\phi_3}\left(\frac{2}{9}\frac{\phi_2^2}{\phi_3} - 1\right) < 0 \end{aligned} \tag{12}$$

The parametrization in (a, b) is in one-to-one correspondence with (ϕ_2, ϕ_3) under the constraints in (7) and (12). The construction of the model in terms of (a, b) or (ϕ_2, ϕ_3) is observationally equivalent. However it proves to be more convenient to proceed with (a, b).

3.4.3 Inversion of the Price Equation II

With $a > 0$, the unique real root for the inverse of (11) can be written as[5]:

$$y_t = h(\Delta p_t; a, b) = A(\Delta p_t; a, b) + B(\Delta p_t; a, b) \tag{13}$$

where

$$\begin{aligned} A(\Delta p_t; a, b) &= \left[-\frac{b - \Delta p_t}{2} + C(\Delta p_t; a, b)\right]^{\frac{1}{3}} \\ B(\Delta p_t; a, b) &= \left[-\frac{b - \Delta p_t}{2} - C(\Delta p_t; a, b)\right]^{\frac{1}{3}} \end{aligned} \tag{14}$$

and $C(\Delta p_t; a, b)$ is

$$C(\Delta p_t; a, b) = \left[\left(\frac{b - \Delta p_t}{2}\right)^2 + \left(\frac{a}{3}\right)^3\right]^{\frac{1}{2}} \tag{15}$$

3.4.4 Conditional Density of the Differenced Price $(\Delta p_t | Z_t)$

Since y_t is a linear transformation of E_t, which is itself a linear function of Z_t, we can transform (6) into:

$$y_t = c' Z_t + \epsilon_t \qquad \epsilon_t \sim N(0, \sigma^2) \tag{16}$$

where c and σ^2 can be obtained by $c = p\gamma$ and $\sigma^2 = p^2 \sigma_v^2$. We will write down the likelihood function and conduct a joint specification search in terms of $\theta = (a, b, c, \sigma^2)$. Once the search is concluded. we should retrieve the original parameters $(\phi_2, \phi_3, \gamma, \sigma_v^2)$ from the estimated parameter values for $\hat{\theta} = (\hat{a}, \hat{b}, \hat{c}, \hat{\sigma}^2)$. As will be explained below. this is not a completely trivial task.

Using (16), we have the following density function for the transformed variable y_t:

$$f(y_t | Z_t; \theta) \propto \frac{1}{\sigma} \exp\left[-\frac{1}{2\sigma^2}(y_t - c' Z_t)^2\right] \tag{17}$$

Next we can apply the inverse transformation from y_t to Δp_t as given in (13) with Jacobian:

[5] This solution is obtained by the Cardano's method. There are alternative ways of obtaining the real root.

$$J(\Delta p_t; a, b) = \frac{\partial h(\Delta p_t; a, b)}{\partial \Delta p_t} > 0 \tag{18}$$

to obtain the density for the Δp_t:

$$f(\Delta p_t | Z_t; \theta) \propto \frac{J(\Delta p_t; a, b)}{\sigma} \exp\left[-\frac{1}{2\sigma^2}(h(\Delta p_t; a, b) - c' Z_t)^2\right] \tag{19}$$

3.4.5 Log-Likelihood Function of the Model

Let

$$\dot{p}(a,b) = \begin{pmatrix} h(\Delta p_1; a, b) \\ \vdots \\ \vdots \\ h(\Delta p_T; a, b) \end{pmatrix} \qquad Z = \begin{pmatrix} Z_1 \\ \vdots \\ \vdots \\ Z_T \end{pmatrix}$$

The log-likelihood function is given by:

$$\ln L(\theta) = -\frac{T}{2}\ln \sigma^2 + \sum_{t=1}^{T} J(\Delta p_t; a, b) - \frac{1}{2\sigma^2}(\dot{p}(a,b) - Zc')(\dot{p}(a,b) - Zc) \tag{20}$$

The log-likelihood function can be maximized analytically in (c, σ^2) given (a, b) by OLS estimation as follows:

$$\tilde{c}(a,b) = (Z'Z)^{-1} Z' \dot{p}(a,b) \tag{21}$$

$$\tilde{\sigma}^2(a,b) = \frac{1}{T}\dot{p}(a,b)' M_Z \dot{p}(a,b) \tag{22}$$

and

$$M_Z = I - Z(Z'Z)^{-1}Z' \tag{23}$$

Ignoring additive constants, the corresponding concentrated log-likelihood is given by:

$$\ln L_c(a,b) = -\frac{T}{2}\ln\left[\hat{\sigma}^2(a,b)\right] + \sum_{t=1}^{T} J(\Delta p_t; a, b) \tag{24}$$

which will be maximized numerically in (a, b).

3.4.6 Jacobian of the Transformation

The Jacobian can be obtained from equations (13), (14) and (15) For ease of notation we ignore the arguments of the related functions (i.e.: $A_t = A(\Delta p_t; a, b)$). Therefore the Jacobian is:

$$J_t = \ln(3\Delta p_t^2 + a) \tag{25}$$

3.4.7 Maximization

The computation of the maximization of log-likelihood is programmed in Gauss programming language and conducted in a Intel Core-7 laptop computer. A single calculation of the optimum parameters given Z_t while depending on the number of iterations and the size of Z_t approximately takes 1-5 minutes on average. The procedure calculates the optimum parameters $(\hat{a}, \hat{b})$ as a result of the values that maximize the concentrated log-likelihood given in equation (24). Once we obtain the ML estimates of $(\hat{a}, \hat{b})$, they are substituted in equations (21) and (22) to obtain the ML estimates of (c, σ^2) as $\hat{c} = \tilde{c}(\hat{a}, \hat{b})$ and $\hat{\sigma}^2 = \tilde{\sigma}^2(\hat{a}, \hat{b})$.

3.4.8 Retrieving the Original Parameters

y_t represents a linear transformation of the latent variable E_t. Easiest way to retrieve (ϕ_2, ϕ_3) from (a, b) is to introduce the inverse transformation

$$y_t = pE_t + q \tag{26}$$

and find (ϕ_2, ϕ_3) as a function of (a, b). Substituting (26) into (11) and imposing the constraints in (7) produces the following:

$$b + aq + q^3 = 0 \tag{27}$$

with $a > 0$, this produces a unique real root $q > 0$. Using this real valued solution, we can solve for p and from that we can obtain (ϕ_2, ϕ_3):

$$\begin{aligned} p &= \frac{1}{a + 3q^2} > 0 \\ \phi_2 &= 3p^2 q \qquad \phi_3 = p^3 \end{aligned} \tag{28}$$

which corresponds effectively to the inverse of (10) as a function of (a, b).

4 Data

We use data from the Istanbul Energy Exchange (EXIST) for the Turkish hourly electricity prices and trade volumes between 2010 and 2015[6]. The electricity prices are for each hour and recorded from 12:00 A.M. to 11:00 P.M. in 24 increments. We use the hourly prices for 00:00 A.M., 08:00 A.M., 11:00 A.M., 02:00 P.M., 06:00 P.M. and 09:00 P.M., and estimate the excess demand models for each of these series separately Table 2 reports the descriptive statistics of the data set used in the analysis. The price exhibits substantial variation during the day, with different price levels are observed at particular hours. This is expected since the price is actually formed in the previous day by a price matching mechanism that takes into consideration the bid and ask prices raised by the suppliers and the distributors. Therefore different levels are mainly related to the cost of producing electricity at different parts of the day. However, we also observe higher price variation during the hours where the price level is high. The standard deviation of the price peaks at 11:00 A.M. and continues to be high at 14:00 P.M. and gradually decreases during the later hours in the day through midnight. The log price change series have mean zero on average and exhibit a standard deviation that is higher for the morning hours than the rest of the day.

Table 2: Descriptive Statistics

Hour	Price		Log Price Change		Log Volume Change	
	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
00:00 A.M.	145.75	28.09	0.00046	0.1337	0.00173	0.3402
08:00 A.M.	153.13	33.78	-0.01302	0.1682	-0.02540	0.4333
11:00 A.M.	174.38	38.20	-0.00044	0.1471	-0.00173	0.4691
02:00 P.M.	168.48	36.35	-0.00495	0.1451	-0.01445	0.4334
06:00 P.M.	153.79	37.42	-0.00265	0.1466	-0.00554	0.3764
09:00 P.M.	146.53	31.64	-0.00032	0.1371	0.00069	0.3370

The time period used is January 2010 to January 2016. Price is the price for one MGW of electricity reported in Turkish Lira.

The price reaction during the day does not follow a linear trend. Figure 2 shows the change in the volume from the previous day drawn against the change in the price. On the top left picture, the result for the midnight is given. The reaction seems to be depicted by a linear reaction well for this hour. Similarly for 8 A.M. in the morning the electricity price change and the change in the volume seems to be captured by a linear

[6] Including first month of 2016.

reaction well again. However for 11:00 A.M. and 2:00 P.M. given in the mid part of the Figure 2, we observe that the reaction of the price starts to diverge from linearity. The same kind of trend to a lesser degree can be observed in the graphs given in the bottom panel for the hours 6:00 P.M. and 09:00 P.M.

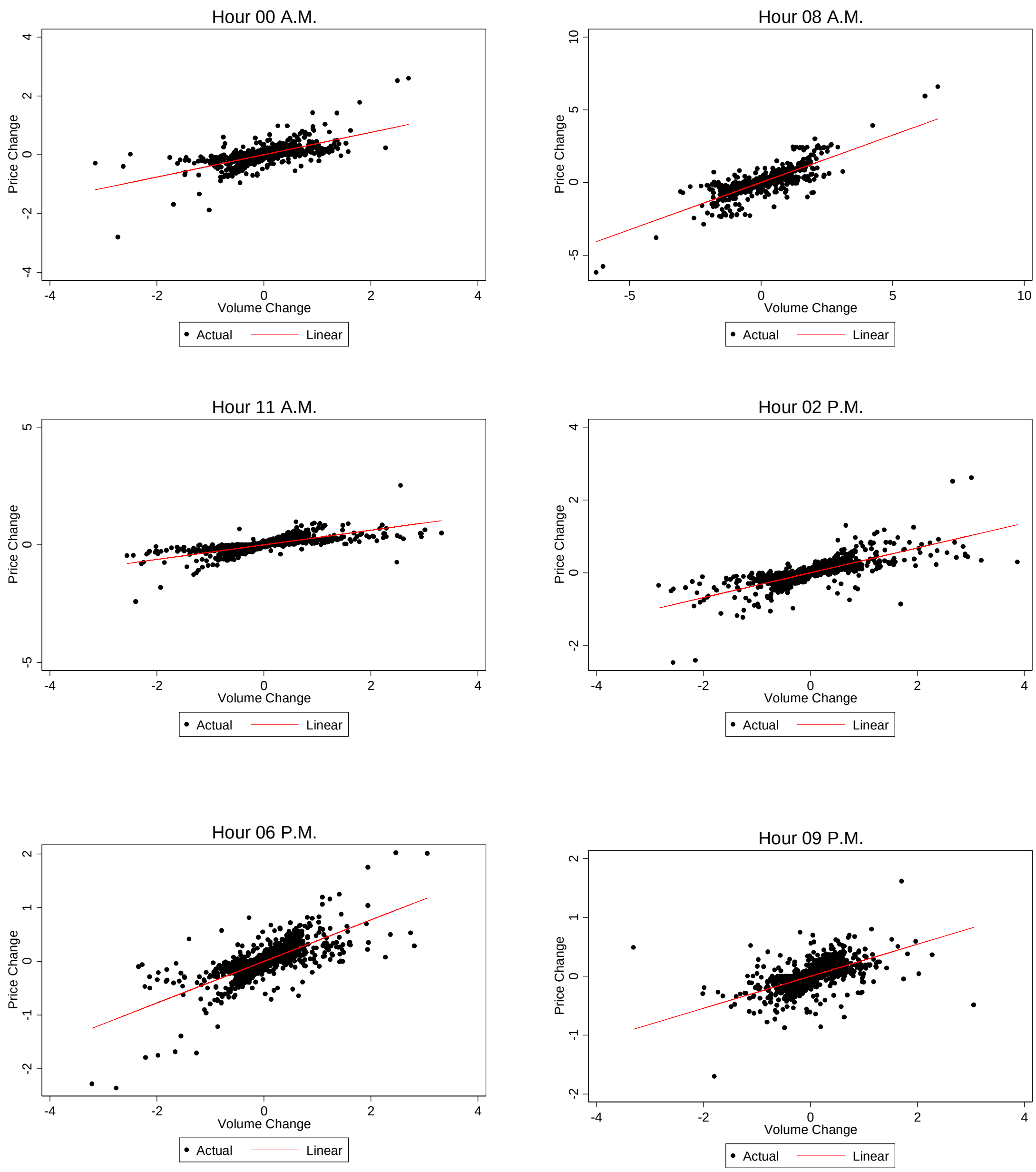

Figure 2: Nonlinearity in reaction

Also the evening hours (called as the puant hours) are the time periods where the deviation from the linearity is more severe. This is of course partly related to the fact that magnitude of change in the volume is not exogenous. The relative demand changes occurs in the periods where the demand is already high.

5 Estimation Results

All of the variables used in the estimation are constructed from the main variables reported in Table 1. The dynamic relationship between the price and the latent excess demand is modeled through the explanatory variables used in the excess demand specification. The excess demand dynamics is captured by including the lags of the reported variables. We are aiming to form a prediction model for the price changes. The dependent variable is the first differenced log prices. The variables used for prediction includes 10 lags of the dependent variable, 5 lags of the change in the volume, and the first differenced log prices of the nearest 10 hours from the previous day. For instance, for the hour 02 P.M., the differenced log prices of hours: 01:00 P.M., 12:00 P.M., 11:00 A.M., 10:00 A.M. and 09:00 A.M. are included as the lag effects and, 03:00 P.M., 04:00 P.M., 05:00 P.M., 06:00 P.M. and 07:00 P.M. are included as the lead effects from the previous day. We collect the right hand side variables in a big vector of Z_t where, Z_t has 42 variables in it including a constant and dummy variables for the day of the week with Monday as the excluded class.

The estimation results are given in Table 3 and 4. We immediately see from the log-likelihoods that the nonlinear model fits the data much better. For all hours, the log-likelihood values that are reported in the first row of Table 3 and 4 are considerably higher than the linear model results (under the assumption of normal errors). To understand the causes for the better performance of the nonlinear model, we report the RMSE in the second row. The RMSE results indicate that, the predicted values for the price changes from the excess demand model are very close to those obtained from the linear model, however our predictive confidence intervals are quite different. In particular, confidence intervals exhibit significant heteroskedasticity with greater uncertainty in the particular periods, where the price volatility is higher. Furthermore we report, AIC, BIC statistics in rows three and four. The nonlinear model is performing better according to this criteria as well. Finally the R^2 measure from the linear model and the pseudo-R^2 from the nonlinear excess demand model is reported in the final row[7] in the top panel in Table 3. In all hour specifications, the nonlinear model produces a better fit measure[8].

[7]Since we use the same dependent variable Δp_t for both linear and nonlinear models, this measure is comparable across models.

[8]For robustness check, we estimated other alternative models employed in the emprical literature

The estimated parameter values from the nonlinear model are given in the second panels of Table 3 and Table 4. In the Tables, the coefficient estimates from the transformed model (a, b) are reported first. Following those estimates, the coefficient estimates for the main parameters of interest, ϕ_2 and ϕ_3 are reported. We find significant nonlinearity where the coefficient ϕ_3 of the cubic term E_t^3 in equation (5) is large for all hours. However, the coefficient estimate for ϕ_2 is close to zero in all specifications. This, we interpret as the symmetric response behavior of the price to the change in the excess demand. The rest of the tables report the estimate for the conversion factor p, estimates for the coefficients c and σ^2 of the transformed model along with the original models' coefficients (γ, σ_v^2) As explained in the model section, the derived coefficients $(\hat{a}, \hat{b}, \hat{c}, \hat{\sigma}^2)$ from the transformed model are estimated using the likelihood function based on the search for these parameters. The original coefficients are then retrieved using the conversion factor p and solving equation (27) for the unique root.

Next we consider the predictive performance of the nonlinear model. It is important to note that we expect the mean prediction performance of the nonlinear model not to differ much from the linear counterpart[9], while the predictive confidence intervals can vary, therefore accounting the heteroskedasticity in the price process. It is especially important to account for the volatility in the short term price prediction in the electricity market where the price can exhibit substantial heteroskedasticity.

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including AR/ARMA models which explain the data with its own dynamics, also linear ARDL model and non linear Markov Switching Regrssion models. For all models our non-linear model has been found superior according to AIC, BIC, Pseudo R^2 and log likelihood values.

[9]In this paper, we are not interested in the selection process for the explanatory variables in the linear (OLS) and the nonlinear models. The same set of explanatory variables are used in the Z_t vector for both specifications. In general, it is not trivial to choose the model variables in a nonlinear model. The model is needed to be estimated for the each new specification again. Given that restriction, researchers often rely on some set of initial explanatory variables from a simplified model first to estimate the nonlinear model. The most appealing candidate is the OLS estimator's explanatory variables which proves to be significant in a simple OLS estimation. This basic idea however misses the opportunity of nonlinear dynamics that are not significant and as a result not captured in the OLS, but may happen to be significant and important in the nonlinear specification. Those considerations in a unified framework for the choice of set of explanatory variables in a recursive procedure which could be potentially different than a linear model are addressed in a subsequent paper.

Also there are various alternatives for the distribution of the stochastic component v_t in equation (6). We assume v_t is distributed normal as $(0, v)$.

Table 3: Estimation Results I

	Hour = 00 A.M.		Hour = 08 A.M.		Hour = 11 A.M.	
	Non-linear	Linear	Non-linear	Linear	Non-linear	Linear
Log Likelihood	2525.39	1606.56	2141.55	1319.92	3489.82	1922.26
RMSE	0.1130	0.1144	0.1223	0.1217	0.1032	0.0995
AIC	-4960.79	-3127.11	-4193.11	-2553.85	-6889.63	-3758.53
BIC	-4706.25	-2883.89	-3943.64	-2315.46	-6634.59	-3514.82
Pseudo R^2	0.2925	0.2683	0.5625	0.4965	0.6660	0.5411

Estimated parameter values of the excess demand (Non-linear) model						
	a	b	a	b	a	b
	0.26766	-1.00E-13	0.2739	-1.00E-13	0.16175	-2.88E-04
	ϕ_2	ϕ_3	ϕ_2	ϕ_3	ϕ_2	ϕ_3
	1.56E-11	52.149	1.46E-11	48.665	0.204	236.276
p		3.73606		3.65095		6.18215

Z_t	c	γ	c	γ	c	γ
constant	-0.15637	-0.04185	0.35318	0.09674	0.30561	0.04943
z_1	-0.00448	-0.00120	-0.02996	-0.00820	-0.07245	-0.01172
z_2	-0.01769	-0.00473	-0.00323	-0.00089	0.01730	0.00280
z_3	-0.00675	-0.00181	-0.01795	-0.00492	-0.01479	-0.00239
z_4	-0.01304	-0.00349	-0.00570	-0.00156	-0.00697	-0.00113
z_5	-0.01553	-0.00416	-0.00505	-0.00138	-0.02878	-0.00465
z_6	-0.14017	-0.03752	-0.12809	-0.03508	-0.12131	-0.01962
z_7	-0.11015	-0.02948	-0.12772	-0.03498	-0.10094	-0.01633
z_8	-0.08319	-0.02227	-0.12232	-0.03350	-0.06599	-0.01067
z_9	-0.06601	-0.01767	-0.12581	-0.03446	-0.05305	-0.00858
z_{10}	-0.03825	-0.01024	-0.09736	-0.02667	-0.03383	-0.00547
z_{11}	-0.04277	-0.01145	-0.06662	-0.01825	-0.04605	-0.00745
z_{12}	-0.02201	-0.00589	-0.02979	-0.00816	-0.01678	-0.00271
z_{13}	-0.02180	-0.00584	-0.03076	-0.00843	-0.02039	-0.00330
z_{14}	-0.01359	-0.00364	-0.01321	-0.00362	-0.01768	-0.00286
z_{15}	-0.01648	-0.00441	-0.01226	-0.00336	-0.01275	-0.00206
z_{16}	-0.00071	-0.00019	0.00045	0.00012	-0.00319	-0.00052
z_{17}	-0.00731	-0.00196	0.05086	0.01393	0.02874	0.00465
z_{18}	0.00146	0.00039	-0.01109	-0.00304	-0.00308	-0.00050
z_{19}	0.01690	0.00452	0.01517	0.00415	-0.02455	-0.00397
z_{20}	-0.01095	-0.00293	-0.01301	-0.00356	-0.00895	-0.00145
z_{21}	0.01468	0.00393	0.02266	0.00621	-0.01083	-0.00175
z_{22}	-0.00042	-0.00011	-0.01912	-0.00524	-0.01401	-0.00227
z_{23}	-0.04342	-0.01162	-0.01905	-0.00522	0.02111	0.00341
z_{24}	0.05620	0.01504	-0.01554	-0.00426	0.02090	0.00338
z_{25}	-0.00243	-0.00065	0.02816	0.00771	-0.00232	-0.00037
z_{26}	-0.00531	-0.00142	-0.00226	-0.00062	0.00141	0.00023
z_{27}	0.00893	0.00239	-0.04132	-0.01132	-0.02330	-0.00377
z_{28}	-0.02982	-0.00798	-0.00152	-0.00042	0.01614	0.00261
z_{29}	0.02104	0.00563	0.00180	0.00049	0.01645	0.00266
z_{30}	-0.00081	-0.00022	0.00912	0.00250	0.01286	0.00208
z_{31}	-0.00536	-0.00143	-0.02521	-0.00691	-0.01520	-0.00246
z_{32}	-0.00146	-0.00039	-0.03441	-0.00942	0.04052	0.00655
z_{33}	0.04714	0.01262	0.04357	0.01193	0.02510	0.00406
z_{34}	-0.03201	-0.00857	0.02722	0.00746	-0.00937	-0.00152
z_{35}	-0.00847	-0.00227	0.01783	0.00488	0.03696	0.00598
z_{36}	0.25464	0.06816	-0.25186	-0.06899	-0.26896	-0.04351
z_{37}	0.13712	0.03670	-0.36802	-0.10080	-0.29486	-0.04770
z_{38}	0.17820	0.04770	-0.35286	-0.09665	-0.27664	-0.04475
z_{39}	0.15560	0.04165	-0.35155	-0.09629	-0.32287	-0.05223
z_{40}	0.21711	0.05811	-0.52812	-0.14465	-0.33880	-0.05480
z_{41}	0.15248	0.04081	-0.80988	-0.22183	-0.67130	-0.10859
	σ	σ_v	σ	σ_v	σ	σ_v
	0.05617	0.00402	0.05257	0.00394	0.05256	0.00138

Table 4: Estimation Results II

	Hour = 02 P.M.		Hour = 06 P.M.		Hour = 09 P.M.	
	Non-linear	Linear	Non-linear	Linear	Non-linear	Linear
Log Likelihood	3495.57	1761.47	2438.29	1458.77	2601.04	1531.61
RMSE	0.1096	0.1073	0.1235	0.1241	0.1198	0.1207
AIC	-6901.13	-3436.93	-4786.59	-2831.54	-5112.08	-2977.22
BIC	-6646.07	-3193.20	-4531.23	-2587.54	-4856.35	-2732.86
Pseudo R^2	0.5652	0.4586	0.3340	0.3017	0.2627	0.2383
Estimated parameter values of the excess demand (Non-linear) model						
	a	b	a	b	a	b
	0.14393	-2.62E-04	0.26236	-3.55E-04	0.23176	-1.00E-13
	ϕ_2	ϕ_3	ϕ_2	ϕ_3	ϕ_2	ϕ_3
	0.263	335.274	0.059	55.366	2.41E-11	80.323
p		6.94704		3.81137		4.31466
Z_t	c	γ	c	γ	c	γ
constant	0.33127	0.04768	0.20274	0.05319	0.12104	0.02805
z_1	0.01686	0.00243	-0.02272	-0.00596	0.00642	0.00149
z_2	0.00871	0.00125	-0.01401	-0.00367	0.01842	0.00427
z_3	-0.00236	-0.00034	-0.01943	-0.00510	0.00161	0.00037
z_4	0.01181	0.00170	-0.00065	-0.00017	-0.00610	-0.00141
z_5	-0.00932	-0.00134	-0.00034	-0.00009	0.00237	0.00055
z_6	-0.15127	-0.02177	-0.12166	-0.03192	-0.15993	-0.03707
z_7	-0.09066	-0.01305	-0.08057	-0.02114	-0.11841	-0.02744
z_8	-0.08175	-0.01177	-0.05924	-0.01554	-0.10053	-0.02330
z_9	-0.08121	-0.01169	-0.06398	-0.01679	-0.06913	-0.01602
z_{10}	-0.05909	-0.00851	-0.05661	-0.01485	-0.04895	-0.01134
z_{11}	-0.05846	-0.00842	-0.04822	-0.01265	-0.03884	-0.00900
z_{12}	-0.02613	-0.00376	-0.00165	-0.00043	-0.02929	-0.00679
z_{13}	-0.02911	-0.00419	-0.01151	-0.00302	-0.02609	-0.00605
z_{14}	-0.01107	-0.00159	-0.00671	-0.00176	-0.00715	-0.00166
z_{15}	-0.01902	-0.00274	-0.01729	-0.00454	-0.01643	-0.00381
z_{16}	-0.02877	-0.00414	0.02185	0.00573	0.01692	0.00392
z_{17}	0.00859	0.00124	0.01317	0.00346	-0.00539	-0.00125
z_{18}	-0.01589	-0.00229	0.01199	0.00314	0.00364	0.00084
z_{19}	0.02285	0.00329	-0.03121	-0.00819	-0.00255	-0.00059
z_{20}	0.00273	0.00039	-0.01098	-0.00288	0.00200	0.00046
z_{21}	0.02492	0.00359	0.01987	0.00521	-0.01359	-0.00315
z_{22}	0.03601	0.00518	-0.00113	-0.00030	0.00737	0.00171
z_{23}	-0.00350	-0.00050	-0.00342	-0.00090	-0.01032	-0.00239
z_{24}	-0.04151	-0.00598	0.00540	0.00142	0.01679	0.00389
z_{25}	0.04833	0.00696	-0.01702	-0.00447	-0.00873	-0.00202
z_{26}	0.06233	0.00897	0.01569	0.00412	-0.00029	-0.00007
z_{27}	-0.01873	-0.00270	-0.01573	-0.00413	0.01171	0.00271
z_{28}	0.00636	0.00091	-0.02198	-0.00577	-0.00082	-0.00019
z_{29}	-0.03644	-0.00525	0.07229	0.01897	0.00113	0.00026
z_{30}	-0.00563	-0.00081	-0.01993	-0.00523	0.00618	0.00143
z_{31}	-0.04482	-0.00645	-0.01896	-0.00498	0.01764	0.00409
z_{32}	0.01505	0.00217	0.01563	0.00410	0.00226	0.00052
z_{33}	0.00891	0.00128	-0.00964	-0.00253	-0.00358	-0.00083
z_{34}	0.03228	0.00465	0.00629	0.00165	0.00142	0.00033
z_{35}	-0.01795	-0.00258	0.03374	0.00885	-0.00030	-0.00007
z_{36}	-0.29669	-0.04271	-0.20215	-0.05304	-0.10965	-0.02541
z_{37}	-0.32719	-0.04710	-0.19338	-0.05074	-0.12037	-0.02790
z_{38}	-0.30854	-0.04441	-0.16627	-0.04363	-0.09281	-0.02151
z_{39}	-0.35145	-0.05059	-0.20772	-0.05450	-0.13807	-0.03200
z_{40}	-0.46486	-0.06691	-0.34282	-0.08995	-0.24036	-0.05571
z_{41}	-0.68911	-0.09919	-0.35107	-0.09211	-0.14032	-0.03252
	σ	σ_v	σ	σ_v	σ	σ_v
	0.06351	0.00132	0.06215	0.00428	0.07004	0.00376

5.1 The nonlinear reaction

In Figure 3, the reaction functions from the nonlinear model are displayed. The cubic reaction function is fitted using the parameter estimates from Table 3 and 4. The reaction of the price change to small changes in the excess demand is well captured by the linearity, however as the magnitude of the excess demand gets larger in absolute value, the deviations start to display nonlinearity. This was the main descriptive evidence for a need of nonlinear modelling. One may argue that this nonlinearity is captured by the OLS estimation as the variables used in the estimation can have the respective coefficient values to capture this behavior. However, remember that the nonlinear model is estimated with the same set of variables used in the OLS estimation, yet still it produces a better fit and better forecast performance (will be detailed below). Also the excess demand specification is a convenient way to conceptualize the economic mechanism in the electricity market where the price movements are realized according to the demand mismatch. The nonlinearity is pronounced more in some hours than other, although the nonlinear model fits the data better for any particular hour. For instance the reaction function estimates for the hours 08:00 A.M., 11:00 A.M., 02:00 P.M., 06:00 P.M. graphically seems to better capture the price behavior than the hours 00:00 A.M. and 09:00 P.M..

Figure 3 draws the predicted log price changes from the model in (5). The reaction is symmetric due to the very small coefficient estimated for the squared term in the specification. This is true for all hours, and can be interpreted as the symmetric response of the price change to changes in the excess demand. The black dots in the figure represent the actual log price changes that corresponds to the estimated excess demand on the horizontal axis. As expected, most of the price changes are small and accumulated around the origin. However the main difference from a linear specification is the different characteristics of the larger changes, as a linear model would be over predicting the effect of small changes and underpredicting the larger ones.

The expected value of the log price change, Δp_t will depend on higher moments of the deterministic and stochastic parts of excess demand function in the nonlinear specification and also the conditional variance will exhibit heteroskedasticity with a specific functional form. Those expectations are needed to be derived for the nonlinear model. We show below the specific form of the mean and the variance equations for log price change in the excess demand model.

Excess Demand and Mean Prediction:

The conditional mean of the nonlinear model can be estimated using the below functional form:

$$E(\Delta p_t|Z_t) = \gamma' Z_t + \phi_2(\gamma' Z_t)^2 + \phi_3(\gamma' Z_t)^3 + (\phi_2 + 3\phi_3\gamma' Z_t)\sigma_v^2 \tag{29}$$

where $\gamma' Z_t$ is the expected value of the excess demand, i.e.: $E(E_t \mid Z_t) = \gamma' Z_t$.[10]

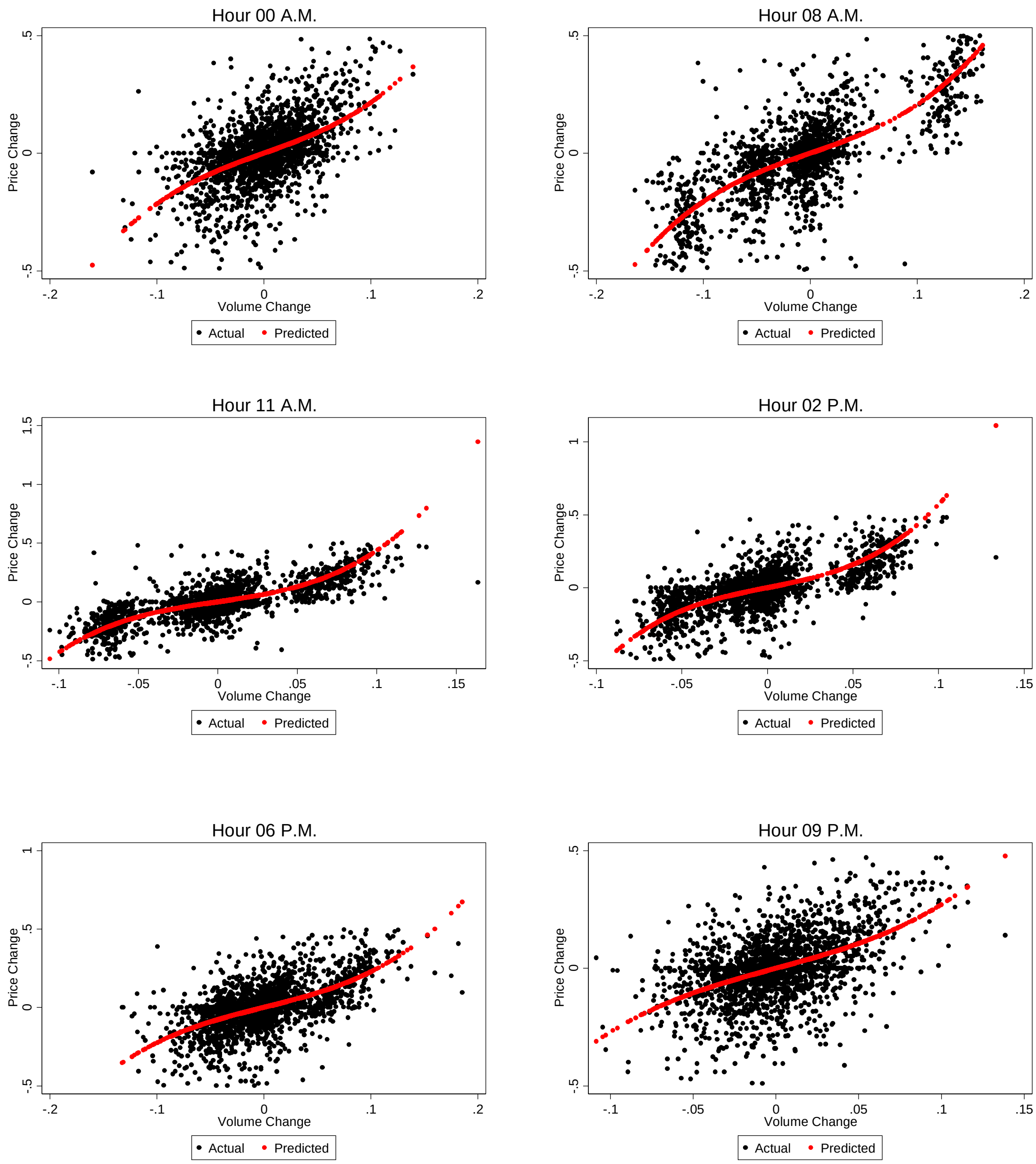

Figure 3: Estimated Reaction Functions

[10]Proof can be found in the Appendix.

Conditional Variance:

The conditional variance can be estimated by the below formula[11]:

$$Var(\Delta p_t|Z_t) = (1+2\phi_2\gamma' Z_t + 3\phi_3(\gamma' Z_t)^2)^2\sigma_v^2 + (2\phi_2^2 + 6\phi_3 + 24\phi_2\phi_3\gamma' Z_t + 36\phi_3^2(\gamma' Z_t)^2)\sigma_v^4 + 15\phi_3^2\sigma_v^6 \quad (30)$$

6 Price Prediction and Confidence Intervals

One of the important outcomes of the model developed and estimated in the previous sections is the capacity to produce price predictions. This is relatively easy with the specification given in equation (29) with all the explanatory variables are in lag forms in Z_t. Therefore at any point in time t, we may produce an estimate for the $t+1$. In this section, we will produce the price predictions from both the linear model and the nonlinear model and also report some statistics regarding the prediction performance. Time series behavior of the latent factor excess demand (E_t) and the conditional variance will be graphically presented along with the price predictions. Note that we did not perform any specification test for the selection of variables in the linear and nonlinear models in this paper and just use the same set of explanatory variables in both. Already reported by the RMSE statistics in Table 3 and 4, the estimated mean price changes are not different among the linear versus nonlinear model, however the nonlinear model specifies a time varying variance for the predictions which is essential in terms of the capturing the increase in the uncertainty in particular periods.

Table 5 shows the performance of the excess demand model in producing price predictions. The analysis is conducted for the whole sample period, and then only for the period from October 2015 to January 2016. In the later period log price changes exhibits more volatility and conditional variance produced by the nonlinear model can be especially important. The first statistics in the table reports the percentage of the time that actual log price change remains in the 1.65 standard deviation band around the predicted price (90% confidence interval). The nonlinear model produces a better catch in this metric, though the difference is not substantial. For the hour 00:00 A.M. for instance, the excess demand model predicts the price 93% of the time in the interval while the OLS estimation predicts 90% of the time. However this difference in the percentages might undermine the performance of the excess demand model since both models perform well. Therefore next we look at the number of times the models miss the actual price change in the prediction interval. This is reported as the third statistics in the table. With this metric for instance, for the hour 00:00 A.M., the

[11] Proof can be found in the Appendix.

excess demand model misses 148 while OLS misses 201 cases in the whole period from January 2010 to January 2016. The same pattern is depicted for the more recent period of October 2015 to January 2016 where only 16 misses happen in the excess demand model compared to 22 from the OLS model.

Table 5: Prediction Results

	Whole Sample					
	Hour = 00 A.M.		Hour = 08 A.M.		Hour = 11 A.M.	
	Nonlinear	Linear	Nonlinear	Linear	Nonlinear	Linear
Power	0.93	0.90	0.92	0.91	0.93	0.92
Expected Loss	0.0788	0.0740	0.0879	0.0758	0.0785	0.0748
Number of misses	148	201	144	162	148	172
N		2066		1712		2076
	Period 2015-10 to 2016-01					
	Nonlinear	Linear	Nonlinear	Linear	Nonlinear	Linear
Power	0.84	0.78	0.73	0.71	0.88	0.91
Expected Loss	0.1009	0.0919	0.1041	0.0871	0.8750	0.9063
Number of misses	16	22	19	20	12	9
N		101		70		96
	Whole Sample					
	Hour = 02 P.M.		Hour = 06 P.M.		Hour = 09 P.M.	
	Nonlinear	Linear	Nonlinear	Linear	Nonlinear	Linear
Power	0.93	0.91	0.93	0.91	0.93	0.90
Expected Loss	0.0828	0.0773	0.0835	0.0824	0.0644	0.0745
Number of misses	140	185	149	196	147	208
N		2076		2098		2141
	Period 2015-10 to 2016-01					
	Nonlinear	Linear	Nonlinear	Linear	Nonlinear	Linear
Power	0.85	0.80	0.87	0.84	0.81	0.67
Expected Loss	0.8476	0.8000	0.8696	0.8435	0.1268	0.1222
Number of misses	16	21	15	18	17	29
N		105		115		88

For the other hours (08:00 A.M., 11:00 A.M., 02:00 P.M., 06:00 P.M. and 09:00 P.M.) similar results are obtained with the exception of the hour 11:00 A.M. only in the most recent period. For this hour in the period from October 2015 to January 2016, the linear model performs better, however the performance of the nonlinear model is superior when the whole sample period from October 2015 to January 2016 is considered. When the whole period is considered, for the 08:00 A.M.,11:00 A.M., 02:00 P.M., 06:00 P.M. and 09:00 P.M., the excess demand model predicts the price in the interval more often than the OLS model parallel with the hour 00:00 A.M. Specifically, excess demand model predicts 92%, 93%, 93%, 93% and 93% of the time in the interval while OLS model predicts 91%, 92%, 91%, 91% and 90%. Also excess demand model misses 144, 148, 140, 149 and 147 cases while OLS model misses 162, 172, 185, 196 and 208 cases for the 08:00 A.M., 11:00 A.M., 02:00 P.M., 06:00 P.M. and 09:00 P.M. respectively.

When the most recent period is considered, for the 08:00 A.M., 02:00 P.M., 06:00 P.M. and 09:00 P.M., the excess demand model predicts better than the OLS model parallel with the results from hour 00:00 A.M.. Excess demand model predicts the price 73%, 85%, 87% and 81% of the time in the interval while OLS model predicts the price 71%, 80%, 84% and 67% of the time. Excess demand model misses 19, 16, 15 and 17 cases while OLS model misses 20, 21, 18 and 29 cases for the 08:00 A.M., 02:00 P.M., 06:00 P.M. and 09:00 P.M. respectively. Also in Table 5, we report a version of an expected loss measure, which is calculated as the average of the absolute value of the deviation of the actual price from the predicted price in the instances where the actual price can not be covered in the 90% confidence interval. The expected loss results are comparable across the linear and nonlinear models.

In Figure 4, the predicted confidence intervals (90%) are presented along with the mean predictions obtained from the excess demand and OLS estimates for the hour 00:00 A.M for illustration. The nonlinear model allows for wider confidence intervals, especially after days following a sudden large price change, the nonlinear reaction function translates that into a higher uncertainty with larger conditional standard deviations. This is obviously limited in the case linear model with a constant variance.

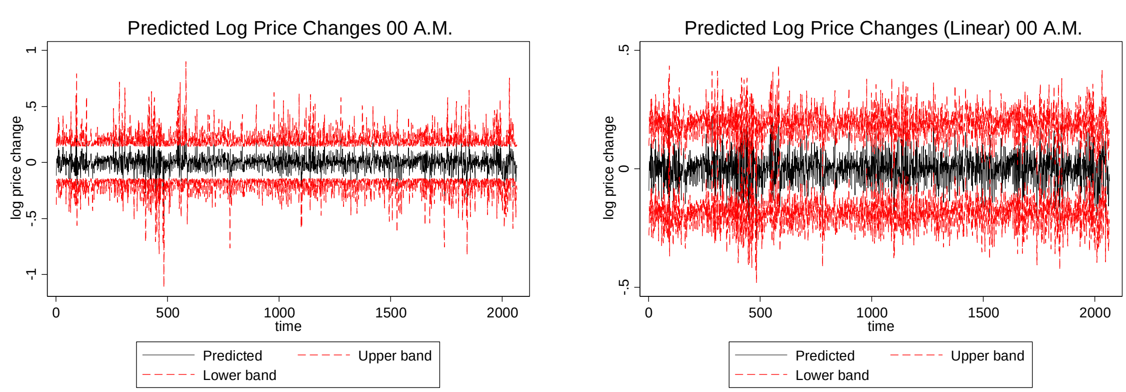

Figure 4: Predicted Confidence Intervals

Figure 5 presents the estimated excess demand, price predictions and the conditional standard deviations from the nonlinear excess demand models for the hours 00:00 A.M., 08:00 A.M. respectively. The price changes in our formulation result from the changes in the excess demand which is a latent variable in the model. We use $\gamma' Z_t$ which is the expected value of the excess demand for time t to construct the first plot. Second graph illustrates the predicted price change. The expected value for the price change is obtained by the formula given in equation (29). Third graph presents the conditional standard deviation estimated from the excess demand model. Equation (30) is used to construct this series and it is plotted with the OLS standard error in Figure 4. Since OLS estimates a constant error variance, the standard deviation from OLS is plotted as the horizontal line in the figure. The last graph presents the model implied price prediction for the price level. The estimated log pice changes are used to form one-day ahead price predictions in this graph. Similarly, Figure 6 presents the estimated excess demand, price predictions and the conditional standard deviations from the model for the hours 11:00 A.M., 02:00 P.M., and Figure 7 presents the estimated excess demand, price predictions and the conditional standard deviations from the model for the hours 06:00 P.M. and 09:00 P.M..

The results show that the estimated mean price changes are not quite different among the linear versus nonlinear model, however the nonlinear model specifies a time varying variance for the predictions which is essential in terms of the capturing the increase in the uncertainty in particular periods. In the figures both the excess demand and relatedly the price predictions exhibit substantial heteroskedasticity. The estimated conditional standard deviations confirm this observation and capture the time varying nature of the volatility process[12]. We can also identify differential characteristics of the excess demand and volatility predictions for the different hours we considered. For some hours the time varying volatility is more pronounced. Therefore understanding the price dynamics will inform us better to characterize the periods as more tranquil and more volatile. Restating again, the mean predictions from the linear and nonlinear models are not so different when the changes in the price are not large, however the predictive confidence intervals are significantly larger under the nonlinear model in times of price instability.

7 Conclusion

This paper proposes a nonlinear stochastic excess demand specification for the electricity prices. The hourly price changes in the Energy Exchange Istanbul (EXIST) are used

[12] Linear models estimated via OLS or AR/RMA models will in general inherently suffer from this heteroskedasticity unless the errors are not specifically adjusted for this.

to estimate the model parameters. It is demonstrated that the nonlinear model fits the data better in terms of log-likelihood, AIC, BIC and pseudo-R^2 criteria. Also model forecasting performance is tested and compared to a linear alternative. The nonlinear model produces a time varying variance which is important in terms of capturing the increase in the uncertainty in volatile periods. However, apart from the model fit, two features of the excess demand model proves to be useful in understanding the short term price dynamics in EXIST, namely (i): the model produces mean predictions for the excess demand (a latent variable in the model) which is economically the reason for the daily price variation; (ii) the implied volatility of the price changes are time varying in the model and they depend on the excess demand as well as the reaction function parameters. This basically brings two important features of time series modelling dynamics together, i.e. the nonlinear mean function, and time varying volatility in a parsimonious model. This paper focuses on the estimation (forecasting) of the nonlinear excess demand model in the spot electricity market, so we are not particularly interested in the selection process for the explanatory variables in the nonlinear model. In nonlinear models, choosing the model variables might not be an easy task in general. For this reason, variable selection sometimes is bypassed by the researchers and an initial set of variables from a simplified model (mostly linear) is relied on. This basic idea however misses the opportunity of nonlinear dynamics that are not significant and as a result not captured for instance in the OLS estimation. Those considerations for the choice of set of explanatory variables in a recursive procedure for the excess demand model are left as future research and currently being addressed in a subsequent paper.

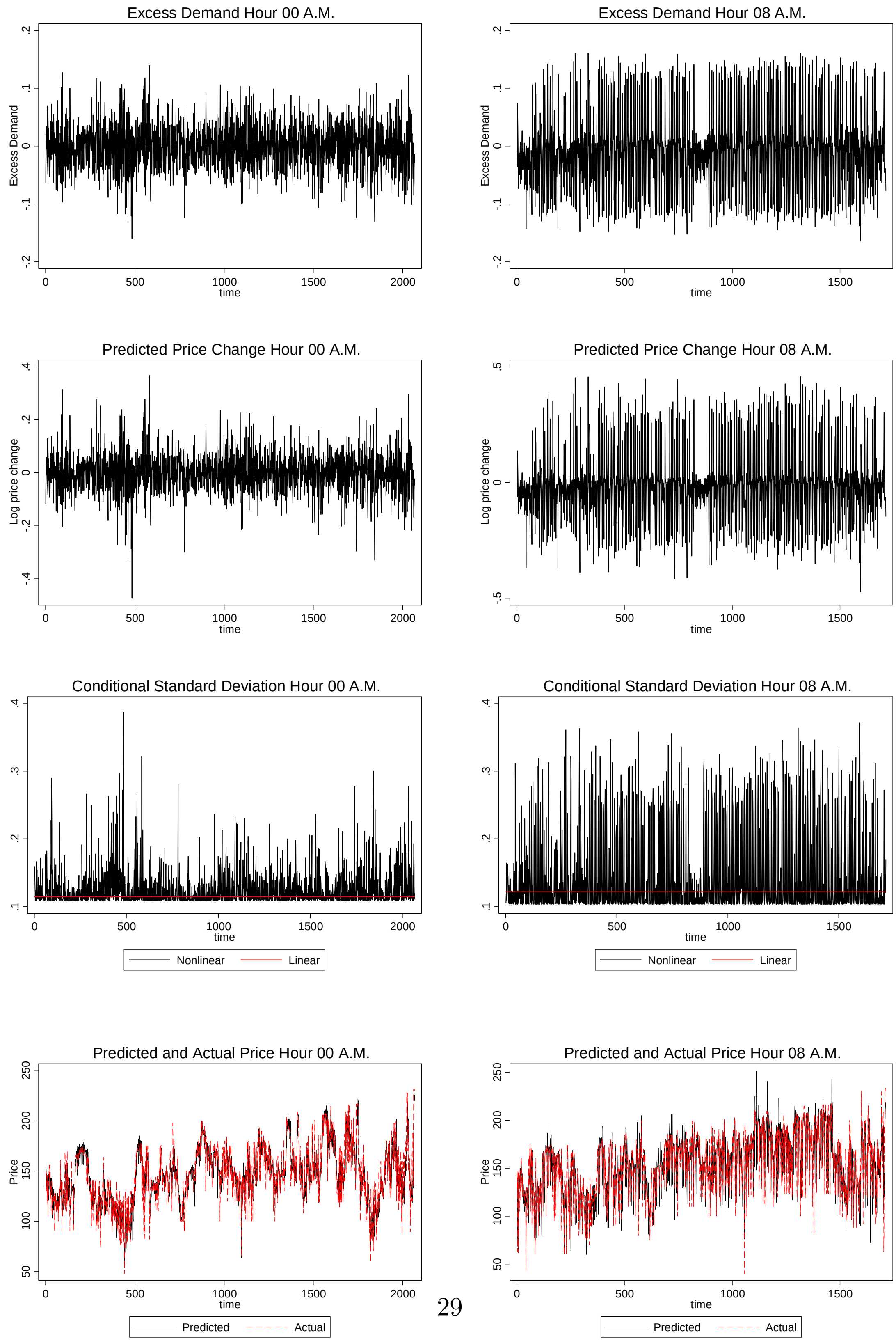

Figure 5: Model Fit, 00 and 08 A.M.

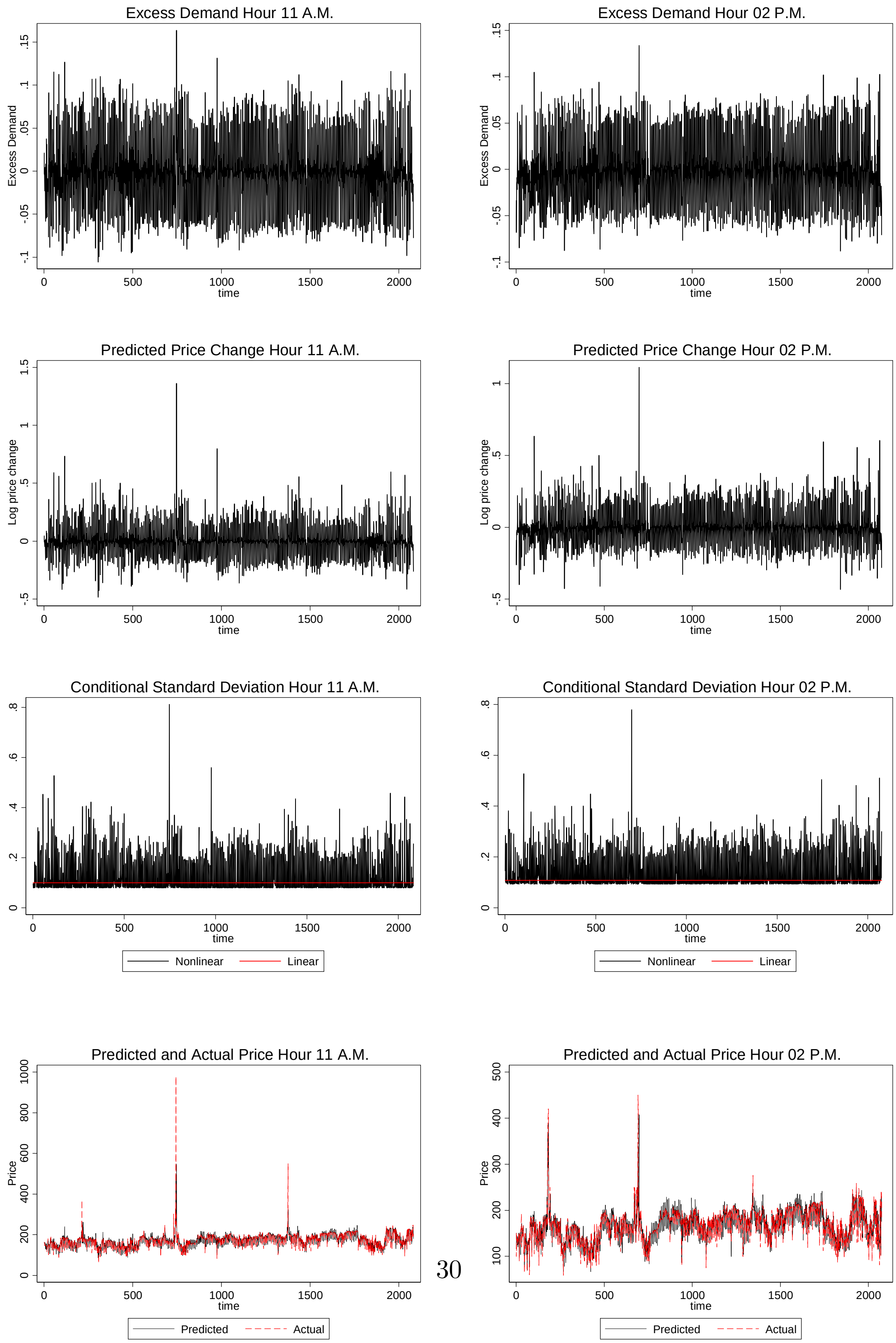

Figure 6: Model Fit: 11 A.M. and 02 P.M.

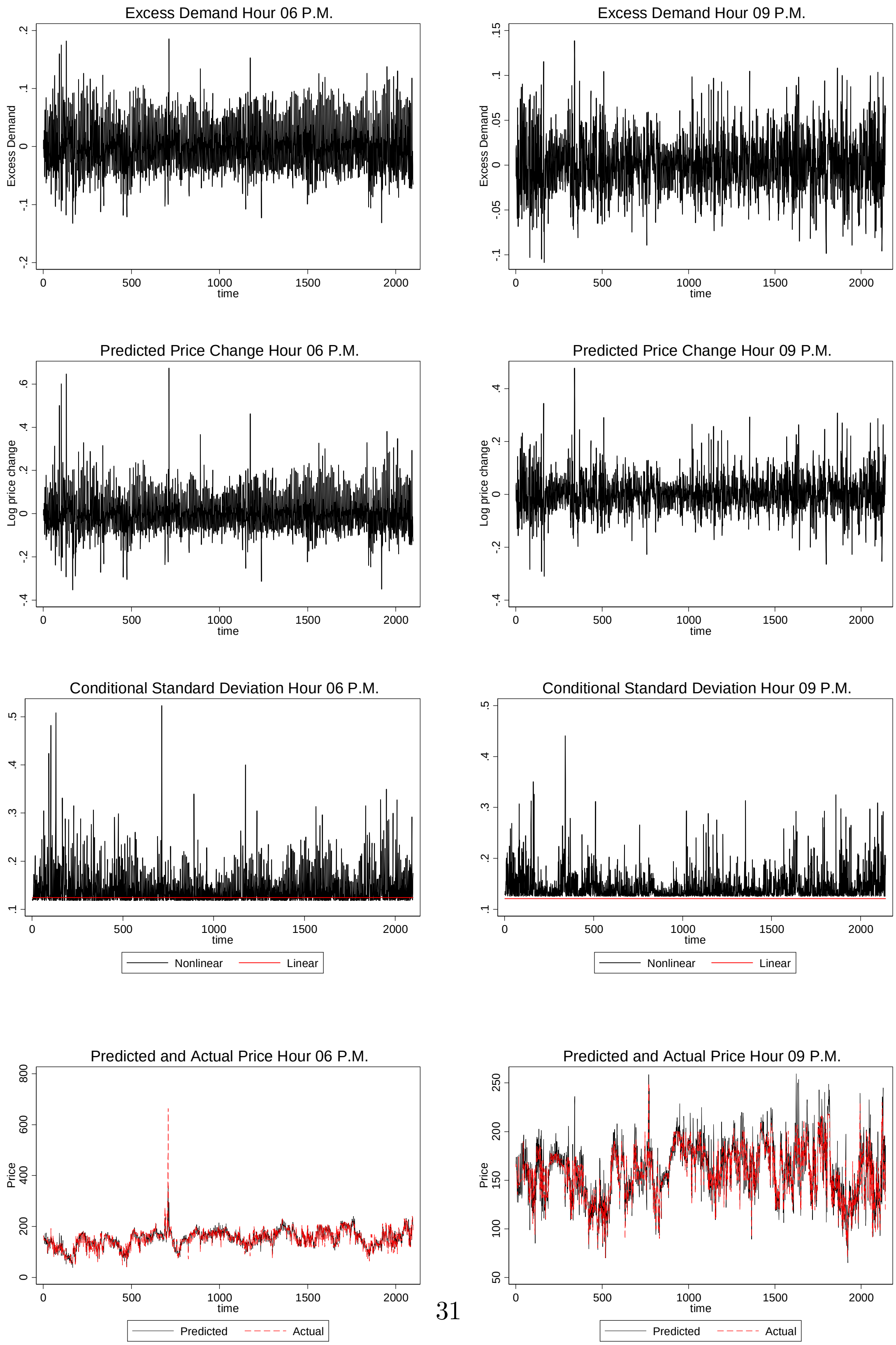

Figure 7: Model Fit, 06 and 09 P.M.

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8 Appendix

Excess Demand and Mean Prediction:
The following expression is the expected price change from the model:

$$\begin{aligned} E(\Delta p_t|Z_t) &= E(E_t + \phi_2 E_t^2 + \phi_3 E_t^3|Z_t) \\ &= E(E_t|Z_t) + \phi_2 E(E_t^2|Z_t) + \phi_3 E(E_t^3|Z_t) \end{aligned}$$

replacing $E_t = \gamma' Z_t + v_t$

$$\begin{aligned} E(\Delta p_t|Z_t) &= E(\gamma' Z_t + v_t|Z_t) + \phi_2 E((\gamma' Z_t + v_t)^2|Z_t) + \phi_3 E((\gamma' Z_t + v_t)^3|Z_t) \\ &= E(\gamma' Z_t|Z_t) + E(v_t|Z_t) + \phi_2 E((\gamma' Z_t)^2|Z_t) + \phi_2 E(2\gamma' Z_t v_t|Z_t) + \phi_2 E(v_t^2|Z_t) \\ &\quad + \phi_3 E((\gamma' Z_t)^3|Z_t) + \phi_3 E(3(\gamma' Z_t)^2 v_t|Z_t) + \phi_3 E(3\gamma' Z_t v_t^2|Z_t) + \phi_3 E(v_t^3|Z_t) \end{aligned}$$

observe that $E(v_t \mid Z_t) = 0$ and $E(v_t^3 \mid Z_t) = 0$ given v_t distributed normal $(0, \sigma_v^2)$ conditional on Z_t. Also $E(v_t^2 \mid Z_t) = \sigma_v^2$. Replacing these into the above expression and taking the conditional expectations on Z_t:

$$\begin{aligned} E(\Delta p_t|Z_t) &= \gamma' Z_t + \phi_2(\gamma' Z_t)^2 + \phi_2 2\gamma' Z_t E(v_t|Z_t) + \phi_2 \sigma_v^2 \\ &\quad + \phi_3(\gamma' Z_t)^3 + 3\phi_3(\gamma' Z_t)^2 E(v_t|Z_t) + 3\phi_3 \gamma' Z_t E(v_t^2|Z_t) \end{aligned}$$

results in:

$$E(\Delta p_t|Z_t) = \gamma' Z_t + \phi_2(\gamma' Z_t)^2 + \phi_3(\gamma' Z_t)^3 + (\phi_2 + 3\phi_3 \gamma' Z_t)\sigma_v^2$$

.

Conditional Variance:
The conditional variance expression for the nonlinear model can be written as follows:

$$\begin{aligned} Var(\Delta p_t|Z_t) &= Var(E_t + \phi_2 E_t^2 + \phi_3 E_t^3|Z_t) \\ &= Var(E_t|Z_t) + \phi_2^2 Var(E_t^2|Z_t) + \phi_3^2 Var(E_t^3|Z_t) + \\ &\quad 2\phi_2 Cov(E_t, E_t^2|Z_t) + 2\phi_3 Cov(E_t, E_t^3|Z_t) + 2\phi_2\phi_3 Cov(E_t^2, E_t^3|Z_t) \end{aligned}$$

replacing $E_t = \gamma' Z_t + v_t$

$$\begin{aligned} Var(\Delta p_t|Z_t) &= Var(\gamma' Z_t + v_t|Z_t) + \phi_2^2 Var((\gamma' Z_t + v_t)^2|Z_t) + \phi_3^2 Var((\gamma' Z_t + v_t)^3|Z_t) + \\ &\quad 2\phi_2 Cov((\gamma' Z_t + v_t), (\gamma' Z_t + v_t)^2|Z_t) + 2\phi_3 Cov((\gamma' Z_t + v_t), (\gamma' Z_t + v_t)^3|Z_t) \\ &\quad + 2\phi_2\phi_3 Cov((\gamma' Z_t + v_t)^2, (\gamma' Z_t + v_t)^3|Z_t) \end{aligned}$$

and deriving the expressions for $Var(.|Z_t)$ and $Cov(.|Z_t)$ for the joint items:

$$\begin{aligned}
Var(\Delta p_t|Z_t) &= Var(v_t|Z_t) + 4\phi_2^2(\gamma' Z_t)^2 Var(v_t|Z_t) + \phi_2^2 Var(v_t^2|Z_t) + 2\phi_2^2(\gamma' Z_t)Cov(v_t, v_t^2|Z_t) \\
&\quad +\phi_3^2[9(\gamma' Z_t)^4 Var(v_t|Z_t) + 9(\gamma' Z_t)^2 Var(v_t^2|Z_t) + Var(v_t^3|Z_t) \\
&\quad +18(\gamma' Z_t)^3 Cov(v_t, v_t^2|Z_t) + 6(\gamma' Z_t)^2 Cov(v_t, v_t^3|Z_t) + 6(\gamma' Z_t)Cov(v_t^2, v_t^3|Z_t)] \\
&\quad +2\phi_2[2\gamma' Z_t Var(v_t|Z_t) + Cov(v_t, v_t^2|Z_t)] \\
&\quad +2\phi_3[3(\gamma' Z_t)^2 Var(v_t|Z_t) + 3(\gamma' Z_t)Cov(v_t, v_t^2|Z_t) + Cov(v_t, v_t^3|Z_t)] \\
&\quad +2\phi_2\phi_3[6(\gamma' Z_t)^3 Var(v_t|Z_t) + 6(\gamma' Z_t)^2 Cov(v_t, v_t^2|Z_t) + 2(\gamma' Z_t)Cov(v_t, v_t^3|Z_t) \\
&\quad +3(\gamma' Z_t)^2 Cov(v_t, v_t^2|Z_t) + 3(\gamma' Z_t)Var(v_t^2|Z_t) + Cov(v_t^2, v_t^3|Z_t)]
\end{aligned}$$

and using the facts $Var(v_t|Z_t) = \sigma_v^2$, $Cov(v_t, v_t^2|Z_t) = 0$, $Var(v_t^2|Z_t) = 2\sigma_v^4$, $Cov(v_t, v_t^3|Z_t) = 3\sigma_v^4$ $Var(v_t^3|Z_t) = 15\sigma_v^6$, $Cov(v_t^2, v_t^3|Z_t) = 0$.

$$\begin{aligned}
Var(\Delta p_t|Z_t) &= \sigma_v^2(1 + 4\phi_2^2(\gamma' Z_t)^2 + 9\phi_3^2(\gamma' Z_t)^4 + 4\phi_2\gamma' Z_t + 6\phi_3(\gamma' Z_t)^2 + 12\phi_2\phi_3(\gamma' Z_t)^3) + \\
&\quad 2\sigma_v^4(9\phi_3^2(\gamma' Z_t)^2 + 6\phi_2\phi_3(\gamma' Z_t) + \phi_2^2) + 3\sigma_v^4(6\phi_3^2(\gamma' Z_t)^2 + 2\phi_3 + 4\phi_2\phi_3(\gamma' Z_t)) \\
&\quad +15\phi_3^2\sigma_v^6
\end{aligned}$$

Rearranging and collecting similar terms yield:

$$\begin{aligned}
Var(\Delta p_t|Z_t) &= (1 + 2\phi_2\gamma' Z_t + 3\phi_3(\gamma' Z_t)^2)^2\sigma_v^2 + (2\phi_2^2 + 6\phi_3 + 24\phi_2\phi_3\gamma' Z_t \\
&\quad +36\phi_3^2(\gamma' Z_t)^2)\sigma_v^4 + 15\phi_3^2\sigma_v^6
\end{aligned}$$

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