between the layers. $\phi_1(f)$ and $\phi_2(f)$ are the phases of the reflection coefficients R for waves incident on the arrays inside the dielectric, as shown in Fig. 2. The left hand side of (1) is the pathlength phase between the two boundaries, while the right hand side is the mean reflectivity phase. These two phase components are plotted against frequency in Fig. 2. The curve with circles is the phase for the dipole array on the dielectric substrate with $\varepsilon_r = 4.56$, at normal incidence. It intersects the pathlength line at a single point only, at about 21 GHz, corresponding to the transmission peak in Fig. 1.

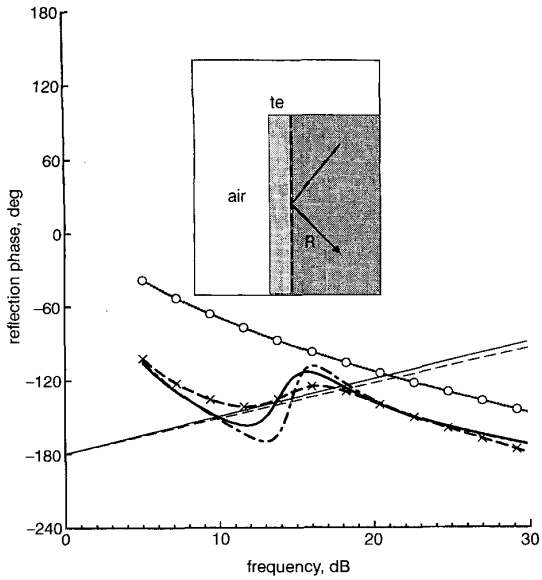

Fig. 2 *Reflectivity phase*
—○— $t_e = 0$, normal incidence
——— $t_e = 3.2$ mm, normal incidence
—·—$t_e = 3.2$ mm, TE45°
—×—$t_e = 3.2$ mm, TM45°

If the profile of the phase curve could be modified so as to intersect the pathlength line at several closely spaced points, or if the two could become coincident over part of their range, the equality in (1) would be satisfied over this range, resulting in a broader transmission band. Introducing additional boundaries into composite FSS structures is a known method for modifying passband shapes [3]. It can also have the required effect on the reflectivity phase, as shown by the three other curves in Fig. 2, which refer to a case in which an external layer of dielectric, of thickness $t_e = 3.2$ mm, is bonded to the outer surface of each dipole array. The curve for normal incidence (solid) now intersects the corresponding solid pathlength line at three points, covering the frequency range 10–18 GHz. Between them it deviates from the line, but not to the extent that the corresponding (broken) curve for 45° TE does. The gradient of the curve for 45° TM does not vary so much over this frequency range, and the phase remains close to the pathlength phase.

This translates into the transmission responses shown in Fig. 3. At normal incidence the −0.5 dB transmission band now extends from about 9 to 20 GHz and the band edge ratio is ~1.1. Therefore this FSS offers octave bandwidths in both transmission and in reflection, with very close bandspacings. There is relatively little drift in the band edges with increasing angle of incidence, but at 45° TE the ripple is more pronounced and the transmittivity falls to ~−1 dB in the two minima. The three peaks correspond to the intersections in Fig. 2. In TM there is almost no ripple in the passband, as would be expected from the phase characteristics in Fig. 2.

The design is not optimised and further improvement is possible, but this example illustrates that the overall performance of these densely packed arrays as FSS can be greatly improved using this technique.

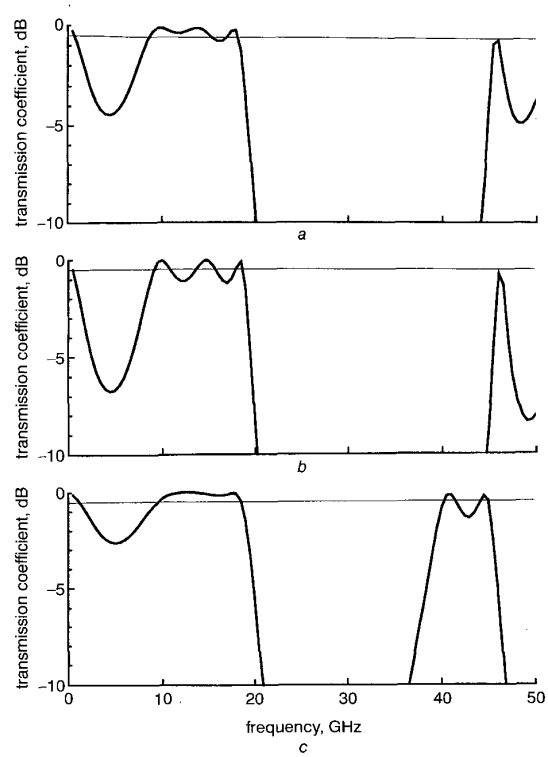

Fig. 3 *Transmission response of double layer FSS with $t_e = 3.2$ mm*
a Normal incidence
b TE45°
c TM45°

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Non-orthogonal space-time block codes for 3TX antennas

M. Uysal and C.N. Georghiades

New non-orthogonal space-time block codes for three transmit antennas are proposed, achieving throughput rates larger than those of currently known orthogonal designs. The proposed codes are found through a code search based on the rank criterion, determinant criterion and rank distribution.

Introduction: Space-time block codes (STBC) are an efficient means of achieving transmit diversity, which are able to achieve the full diversity promised by the multiple-input multiple-output antenna systems [1]. However, they suffer a loss in throughput rate for complex signal constellations such as PSK and QAM. The full-diversity codes given in [1] achieve only a rate of $R = 1/2$ for the complex constellations. Also, some sporadic codes exist for the special case of three and four transmit antennas with $R = 3/4$ [1]. Guaranteeing full diversity requires the orthogonality of the code matrix, on which Tarokh *et al.*'s designs [1] are based. However, by relaxing the orthogonality requirement, at the cost of some performance loss, it is possible to construct new STBCs for real/complex constellations achieving transmission rates larger than those of the current orthogonal designs. Some non-orthogonal codes are proposed in [2–5] for four transmit antennas, based on arbitrary construction. They are able to operate at full-rate and achieve a diversity order of 2. In this Letter, we focus on the case with three transmit antennas and conduct a code search to find non-orthogonal codes which will outperform the orthogonal designs in throughput rate or, equivalently, in bandwidth efficiency.

Code search criteria: An STBC code is defined by a $P \times M$ code matrix, the entries of which are linear combinations of the variables $\pm s_1, \pm s_2, \ldots, \pm s_L$ and their conjugates. Here M denotes the number of transmit antennas and P represents the number of time slots for transmitting L symbols, resulting in a code rate $R = L/P$. STBC design is based on two criteria [1]: the rank criterion and determinant criterion, which determine diversity order and coding gain, respectively. Let $\boldsymbol{H}(e)$ denote a valid transmission matrix chosen erroneously by the maximum likelihood decoder in favour of the originally transmitted matrix $\boldsymbol{H}(c)$. We can then define the Hermitian square of difference matrix, which we will call the 'distance matrix' thereafter, as

$$\boldsymbol{F}(c, e) = (\boldsymbol{H}(c) - \boldsymbol{H}(e))^T(\boldsymbol{H}(c) - \boldsymbol{H}(e)) \tag{1}$$

where $(\cdot)^T$ denotes conjugate transpose. According to the rank criterion, the minimum rank of $\boldsymbol{F}(c, e)$ for any possible pair of $\boldsymbol{H}(c)$ and $\boldsymbol{H}(e)$ determines the maximum diversity gain. In Tarokh *et al.*'s design [1], the columns in the transmission matrix are orthogonal to each other, making the distance matrix full rank also. Therefore, orthogonal designs achieve the full diversity possible. From a bandwidth viewpoint, however, these codes suffer 50% bandwidth loss for complex constellations since they only provide rate $R = 1/2$ when $M \geq 3$ in general. Since we aim to achieve a data rate equal or larger than 1 (i.e. full-rate or over full-rate codes), we cannot impose the orthogonality restriction; therefore we cannot achieve the full diversity. In our case, the diversity order will be restricted to the minimum rank of the distance matrix over all its possible realisations.

The second criterion proposed in [1] is used to determine the coding gain: the determinant of $\boldsymbol{F}(c, e)$ must be maximised over every pair of $\boldsymbol{H}(c)$ and $\boldsymbol{H}(e)$ in order to achieve the most coding advantage. Mathematically speaking, using the matrix determinant inequality [6], we can write

$$|\det(\boldsymbol{F}(c, e))| \leq \prod_{i=1}^{M} \left(\sum_{j=1}^{M} |\boldsymbol{F}_{i,j}(c, e)|^2 \right)^{1/2} \tag{2}$$

When $\boldsymbol{F}(c, e)$ is nonsingular, equality holds if and only if the columns of $\boldsymbol{F}(c, e)$ are orthogonal. Geometrically, $|\det(\boldsymbol{F}(c, e))|$ is the volume of the M-dimensional parallelepiped, the generating edges of which are given by the columns of $\boldsymbol{F}(c, e)$. When the generating edges are orthogonal, this volume takes its largest value and is simply computed as the product of the lengths of the edges. The cross terms of $\boldsymbol{F}(c, e)$ are given as

$$\boldsymbol{F}_{i,j}(c, e) = \sum_{p=1}^{P} (\boldsymbol{H}(c) - \boldsymbol{H}(e))^T_{i,p} (\boldsymbol{H}(c) - \boldsymbol{H}(e))_{p,j}, \quad i \neq j \tag{3}$$

representing the inner-interference induced by the non-orthogonality. The introduction of cross terms due to non-orthogonality causes a decrease in effective distance of the code. Therefore, in our case, the goal should be to find the best non-orthogonal codes in terms of minimised inner-interference. At this point, it is possible to see an analogy between the effect of non-orthogonality and spatial fading correlation [7]. As derived in [7, eqn. 7], the correlation between transmit antennas introduces additional cross terms in the pairwise error probability (PEP) and results in a performance degradation. The same effect arises this time over the independent channels because of the non-orthogonal structure of the code itself, not the channels.

In general, the rank and determinant criteria are sufficient to largely define the performance of space-time codes. These criteria are originally derived under high SNR approximations. However, at low SNR, the rank distribution of error paths may have a significant effect on the overall performance. If the vast majority of the error events have corresponding distance matrices with a rank larger than the minimum rank (which determines the diversity order), the performance will be dominated by these events, before it attains its asymptotic behaviour. Therefore, in code design the rank distribution should also be considered besides the minimum rank.

Search procedure: Based on the criteria summarised in the preceding Section, we conduct an exhaustive search to find new space-time block codes which will support higher data rates than those of presently available orthogonal designs. First we choose a transmission matrix size along with the number of symbols that will be sent in the allocated transmission slots, which together yield the desired code rate to achieve. In the candidate code matrix, we only allow the use of an element, its negative, its conjugate and its negative conjugate (i.e. $\pm s_k$, $\pm s_k^*$) similar to complex designs in [1]. In the search for each candidate, first the rank value and the rank distribution are found, then the candidates are sorted according to the rank value. Among the candidates with the same rank, rank distribution is taken into account. The codes are then sorted based on the inner-interference value. Finally, the code(s) with minimum average inner-interference value is (are) selected.

Search results: In code search, we focus on different matrix sizes, leading to throughput rates equal to or larger than 1. For code matrix size 2×3, we aim to send three symbols in two time slots, therefore, achieving a data rate of R=3/2. This means an over-rate-1 code, unlike the bandwidth-inefficient orthogonal codes. Some of the best codes found in terms of our criteria are

$$\begin{bmatrix} s_1 & s_3 & -s_2 \\ s_2 & -s_1^* & -s_3 \end{bmatrix}, \begin{bmatrix} s_1 & s_3 & -s_2 \\ s_2 & s_1^* & s_3 \end{bmatrix} \tag{4}$$

The above codes achieve two and three times the transmission rates of Tarokh *et al.*'s codes in [1], which are only able to achieve R=1/2 and R=3/4. As another example, we achieve a data rate of R=4/3 by constructing a matrix structure that allows the transmission of three symbols in four time intervals. For this case, some codes satisfying our criteria are

$$\begin{bmatrix} s_1 & s_4 & s_3^* \\ s_2 & s_3^* & s_4^* \\ s_3 & -s_2^* & -s_1^* \end{bmatrix}, \begin{bmatrix} s_1 & s_4 & s_2^* \\ s_2 & s_3^* & -s_1^* \\ s_3 & -s_2^* & -s_4^* \end{bmatrix} \tag{5}$$

Note that the above codes give transmission rates which are 1.778 and 2.667 times those given in [1] for the three transmit antenna case. Finally, we give full-rate (i.e. R=1) codes for the three transmit antenna case. Since these codes operate at full-rate, they do not cause any bandwidth loss. Some of these codes are

$$\begin{bmatrix} s_1 & s_2 & s_3 \\ s_2^* & -s_1^* & s_1 \\ -s_3^* & -s_3^* & s_2^* \end{bmatrix}, \begin{bmatrix} s_1 & s_2 & s_3 \\ -s_2^* & s_1^* & s_1 \\ -s_3^* & -s_3^* & s_2^* \end{bmatrix} \tag{6}$$

Numerical results and conclusion: We have presented simulation results for the new non-orthogonal STBCs over Rayleigh fading channels. Quadrature phase shift keying (QPSK) is used in the simulations as an example for complex constellation. In Fig. 1, we demonstrate symbol error probability results where we compare the new codes to the orthogonal designs denoted by *Tarokh1* and *Tarokh2* [1]. The proposed codes with R=3/2, R=4/3 and R=1 are denoted by *new1*, *new2* and *new3*, respectively. We also include the performance of previously proposed non-orthogonal codes for four transmit antennas for the purpose of comparison. Due to their common quasi-orthogonality, the non-orthogonal codes independently proposed in [3–5] give identical performance although their code structures seem to be different at first sight. The performance

results demonstrate that relaxing the orthogonality requirement results in a performance degradation. The degradation is due to the inner-interference, resulting from the non-orthogonality of the transmission matrix. The proposed codes *new2* and *new3* achieve a diversity order of 2, losing one diversity order. At this point we should emphasise that these codes, unlike the orthogonal designs, do not cause any bandwidth loss and even achieve bandwidth reduction in the case of *new2*. Also, note that in the previous works on non-orthogonal space-time block codes [2–5], this diversity order (i.e. 2) is obtained with codes using four transmit antennas. Therefore our results demonstrate that there is a trade-off between the diversity order and transmission rate and by proper design it is possible to obtain codes with minimum diversity loss. We also point out the interesting behaviour of the proposed code *new1* in the low and medium SNR region. Although this code is only able to achieve a diversity order of 1, it acts as if it provides a diversity order of 2 before it reaches its asymptotic behaviour. This is as a result of its rank distribution's tendency to 2. Since the vast majority of the error events have corresponding distance matrices with rank 2, although the minimum is just 1, the performance is dominated by these events at the smaller SNRs.

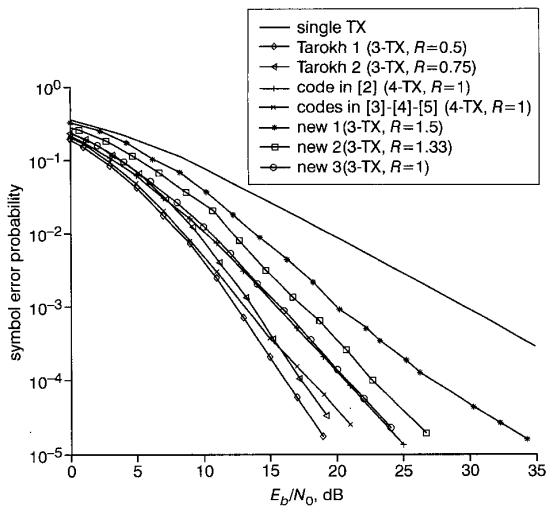

Fig. 1 *Symbol error probability performance of non-orthogonal codes*

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Search for good $b/(b+1)$ high rate recursive systematic convolutional component codes

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A simplified search for good recursive systematic convolution codes with rate $b/(b+1)$ is proposed, to be used as component codes in very high rate, parallel concatenated turbo codes. The first optimised parameter is the effective free-distance of turbo codes. The results obtained are compared to known bounds and previously published results where available.

Introduction: In this Letter we propose the method and the results of a simplified search for good multi-binary systematic recursive convolutional codes of rate $b/(b+1)$ to be used as component codes for parallel concatenated, high rate turbo codes. To achieve high rates in concatenated codes we need higher rate component codes, either multi-binary or punctured. It has been shown [1, 2] that multi-binary codes often have better properties in terms of free-distance (d_f), input weight-2 minimum distance (d_2) and multiplicities than corresponding rate punctured convolutional codes. Moreover, the use of soft-in-soft-out algorithms based on dual reciprocal codes [3, 4] makes these codes soft decodable with low complexity, even at very high rates.

The method described simplifies the search, yielding good component codes and generally includes the optimum code although this is not guaranteed. We have explored memory 3, 4 and 5 codes, for a code rate range from 2/3 up to 10/11. Memory 6 codes have not been considered owing to complexity and convergence issues in concatenated schemes.

Good component codes for concatenated schemes have particular requirements. Generally speaking, the search has to take into account not only the code weight of error events but also their information weight. It is very important that information weight 1 sequences have high code weight, and d_2 is a crucial parameter, even more important than d_f [5]. These issues were addressed in some papers on the same topic [1,2], but with code rate and memory ranges not as wide as presented here. Consideration for the detailed weight spectrum has been neglected in more recent exhaustive searches [6,7] probably because these were aimed at stand-alone high rate convolutional codes.

Simplified search criteria: The basic idea for the simplified search is to start from good memory m, rate $b/(b+1)$ recursive systematic convolutional (RSC) codes defined by a set of feed-forward polynomials $g_1(D), g_2(D), \ldots, g_b(D)$ and feedback $h(D)$, and extend these to $(b+1)/(b+2)$ RSC codes by adding one extra feed-forward polynomial $g_{b+1}(D)$. In this way the search becomes a search for the best single polynomial $g_{b+1}(D)$. Not every choice for $g_{b+1}(D)$ is allowed; to target a good component code we have applied the following constraints:

(i) We use a primitive feedback polynomial $h(D)$. In this way input-weight 1 sequences will have infinite code weight ($d_1 = \infty$) if we avoid using $h(D)$ as $g_{b+1}(D)$.

(ii) We avoid using a generator already in use, unless we are forced to do so when the rate is too high ($b \geq 2^m$). If two generators coincide the distances d_2 and d_f, will be reduced to 2 [7].

(iii) We consider only polynomials with nonzero constant coefficient (odd numbers in octal notation). A $g(D)$ and its shifted version $D^i g(D)$ produce the same single error event spectra up to code weights equal to $2d_f - 1$, for any input weight. Differences in higher code weight terms are marginal.

Good convolutional codes for concatenated schemes: The actual definition of best component code is not trivial. The performance of the final concatenated scheme may be affected more by different parameters of the component codes, depending on the block size, interleaver design and the weight spectrum itself. A well-known fact is that with arbitrary large block size, at moderate to high signal-to-noise ratio (SNR), the performance of the turbo code is dictated by the effective free-distance $d_{free,eff} = 2d_2 - 2$ [5]. Thus d_2 is the parameter that has to be maximised (and its multiplicity N_2 minimised) when the block size is very large. However, depending on the SNR, and with different interleavers and smaller blocks, also error events having information weights 3, 4 and 5 can be critical [8] and can determine