Performance analysis of imperfect closed-loop power control over Rayleigh fading

S. Choe, H.M. Kwon and M. Uysal

The performance of closed-loop power control in code division multiple access systems considering several non-ideal conditions such as multiple power control, group delays, channel estimation error, and power control command bit error probability is investigated.

Introduction: Closed-loop power control (CLPC) is a powerful tool to mitigate near-far problems in the reverse link of a CDMA system over fading channels. Thus far in the literature, most of the work addressing CLPC performance has relied on Monte-Carlo simulations because the feedback loop introduces nonlinearity limiting analytical approaches [1]. Although Song *et al.* present some analytical results in [2], they assume ideal (but impractical) loop conditions, i.e. perfect channel estimation, a single power control group (PCG) delay, and a zero power control command bit (PCB) error probability. In this Letter, we extend the statistically linearisd method in [2] and present a rigorous analysis, including imperfect channel estimation, multiple PCG delays, and a nonzero PCB error probability.

System model: We consider the reverse link of a cellular CDMA system. Fig. 1 illustrates a typical CLPC block diagram under non-ideal conditions. Considering the additional delay of $d_A \geq 1$ PCG due to the processing delay and the round-trip delay [1], the total power control loop delay would be $d \times T_p = (d_A + 1) \times T_p$, where T_p is the PCG period. To represent the PCB transmission error, the PCB is multiplied by a $f_n = \pm 1$ binary random variable with a specified error rate. The additive white Gaussian noise (AWGN) power is much smaller than the interference (intracell and intercell) power Ξ and is neglected. Furthermore, Ξ is assumed to be a stationary process (constant). The channel estimation error (unit dB) is defined as $C_n = G_n - \hat{G}_n$, where G_n is the power attenuation introduced by the fading, and $\hat{G}_n$ is the estimated version. The subscript n denotes the index of the PCG in the CLPC loop.

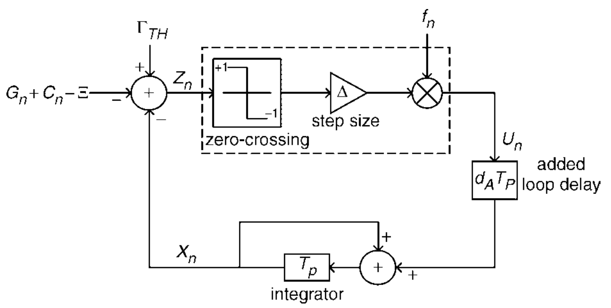

Fig. 1 *Block diagram of closed-loop power control loop*

First, we assume the case of perfect channel estimation, $C_n = 0$ dB, i.e. $G_n = \hat{G}_n$, which later will be used as a benchmark. Let X_n be the transmit power level of a mobile station (MS). For the nth PCG, the base station (BS) measures the received signal-to-interference ratio (SIR) (dB) $\Gamma_n = X_n + G_n - \Xi$ from MS and compares it with the desired SIR threshold ratio Γ_{TH} to generate a single PCB. If $\Gamma_n \geq \Gamma_{TH}$, then the BS transmits a negative PCB for MS to decrease its transmitting power by the power control step size ΔdB for the next PCG interval. Otherwise, a positive PCB is transmitted to increase the power by ΔdB. Therefore, the state equation for the CLPC can be written as:

$$
\begin{aligned}
X_n &= X_{n-1} + U_{n-d} \\
&= X_{n-1} + \Delta\Psi(\Gamma_{TH} - \Gamma_{n-d}) f_{n-d}
\end{aligned}
$$

where U_n is the PCB with values of $\pm\Delta$ (dB), and $\Psi(\cdot)$ denotes the signum function [2].

The nonlinear portion of the CLPC is illustrated by dashed lines in Fig. 1. The power control error Z_n can be modelled as a Gaussian random variable (a lognormal random variable in a linear scale) which has a zero-mean and variance of σ_c^2 [1, 2]. The nonlinear part with a two-level Gaussian quantiser can be well approximated linearly with both a gain component and an additive noise component [2, p. 280]. Hence, the received PCB signal $U_n = \Delta\Psi(Z_n) f_n$ at an MS is given as $\hat{U}_n = gZ_n + W_n$, where g and W_n denote the gain component and the linear approximation error, respectively. W_n is modelled as a Gaussian random variable with zero-mean and variance of σ_W^2 and independent of Z_n. Hereinafter, we drop the subscript n of all random variables for the sake of notational convenience.

The mean square error (MSE) is given by $\mathrm{MSE} = E\{(U - gZ)^2\} = \Delta^2 + g^2\sigma_Z^2 - 2gE\{UZ\}$, where $E\{UZ\} = \Delta E\{Z\Psi(Z)\}E\{f\}$. The optimum g^* (in terms of minimising MSE) is given by $g^* = \sqrt{2/\pi}\Delta(1 - 2p)/\sigma_Z$, where $P(f = -1) = p$ is the PCB error probability. The variance of W is then obtained as

$$
\sigma_W^2 = E\{U^2\} - (g^*)^2 E\{Z^2\} = \Delta^2\left\{1 - \frac{2}{\pi}(1 - 2p)^2\right\}
$$

The input to the CLPC in Fig. 1 is redefined as $Y = \Gamma_{TH} - G + \Xi$ for the convenience of analytical derivation. We can now define the variance of power control error Z as

$$
\sigma_Z^2 = \frac{1}{2\pi}\left\{\int_{-\pi}^{\pi} |H_{Y,Z}(e^{-j\omega})|^2 S_Y(e^{-j\omega}) dw + \sigma_W^2 \int_{-\pi}^{\pi} |H_{W,Z}(e^{-j\omega})|^2 d\omega\right\} \tag{1}
$$

where $S_Y(e^{-j\omega})$ represents the power spectral density of input Y, which follows Jakes' model in our case [3]. Here, the frequency transfer function from the input Y to Z, i.e. $H_{Y,Z}(e^{-jw})$ and the frequency transfer function from the input W to Z $H_{W,Z}(e^{-jw})$ are defined, respectively, as $H_{Y,Z}(e^{-jw}) = (1 - e^{-jw})/(1 - e^{-jw} + g^* e^{-jwd})$ and $H_{W,Z}(e^{-jw}) = -e^{-jwd}/(1 - e^{-jw} + g^* e^{-jwd})$.

Now, we turn our attention to the case of imperfect channel estimation, i.e. $C \neq 0$ dB. Assuming C is i.i.d. with zero-mean, its autocorrelation is given as $R_C[k] = \sigma_C^2 \delta(k)$, where $\delta(\cdot)$ is the delta function. Defining the ratio $\alpha = \sigma_C^2/\sigma_Z^2$, the variance of power control error Z can be derived as

$$
\sigma_Z^2 = \frac{1/2\pi\left\{\int_{-\pi}^{\pi} |H_{Y,Z}(e^{-j\omega})|^2 S_Y(e^{-j\omega}) d\omega + \sigma_W^2 \int_{-\pi}^{\pi} |H_{W,Z}(e^{-j\omega})|^2 d\omega\right\}}{1 - \alpha/2\pi \int_{-\pi}^{\pi} |H_{C,Z}(e^{-j\omega})|^2 d\omega} \tag{2}
$$

where $H_{C,Z}(e^{-j\omega})$ has a similar form as $H_{Y,Z}(e^{-j\omega})$. The power control error variance σ_Z^2 in (1) and (2) can be solved numerically once the channel variation input spectrum $S_Y(e^{-j\omega})$ is known [2].

Results: We consider a wireless system with 2 GHz carrier frequency, 10 Kbit/s data rate, and 2 kHz PCG rate. We adopt the Jakes' fading model with a normalised maximum Doppler frequency, which ranges from $f_D T_b = 0.0037$ (20 km/h) to 0.0185 (100 km/h). We consider a single-path Rayleigh fading channel (i.e. a flat-fading channel) and a two-path Rayleigh fading channel (i.e. a frequency selective fading channel with one tap and equal gain profile) as examples. We assume that a single bit is used to represent the transmitted PCB and that the power control step size Δ is set to 1 dB.

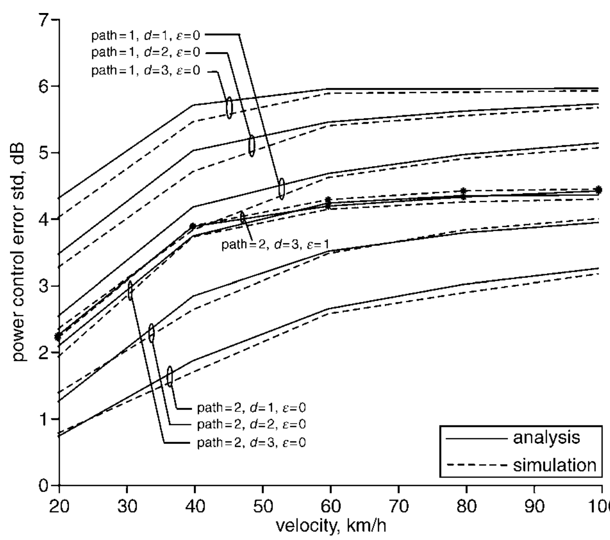

Fig. 2 *Analytical and simulation results of σ_z (dB) for $\alpha = 0$*

Fig. 2 illustrates the standard deviation of the power control error σ_Z (dB) for the case of perfect channel estimation, i.e. $\alpha = 0$. Both analytical and simulation results are given for various loop delays of $d = 1$, 2 and 3 and PCB error rates of $\varepsilon = 0$ and 10%. We observe that the simulation and analytical results show a good match, with a discrepancy of 0.2 dB for a two-path fading channel (path = 2) and a discrepancy of 0.3 dB for a single-path fading channel (path = 1). It is worth noting that the discrepancy gets smaller for the two-path channel, which allows a better Gaussian approximation. Fig. 2 demonstrates that a loop-delay increment of one PCG unit increases the CLPC loop error by 0.4–1.0 dB (The 1 dB increment of power control error standard deviation corresponds to above 30% loss of user capacity [4]). Furthermore, it is observed that the slope of the standard deviation of the CLPC loop error is larger at lower speeds (e.g. 20–40 km/h) than that observed at higher speeds. Finally, it is interesting to note that the increment of the PCB error rate ε from 0 to 10% (see the lines with stars in Fig. 2) does not change the CLPC loop error standard deviation significantly [1].

In Fig. 3, we illustrate the power control error standard deviation σ_Z (dB) for the case of imperfect channel estimation, i.e. $\alpha \neq 0$. The PCB error rate is assumed to be 0%, and the loop delay is taken as $d = 2$, 3. It can be observed from Fig. 3 that the channel estimation error degrades the PC loop performance almost uniformly for an entire range of mobile speeds. Each 10% increase of α increases the standard deviation of the PC loop error by about 0.4 dB for a single-path fading channel and 0.3 dB for a two-path fading channel, respectively.

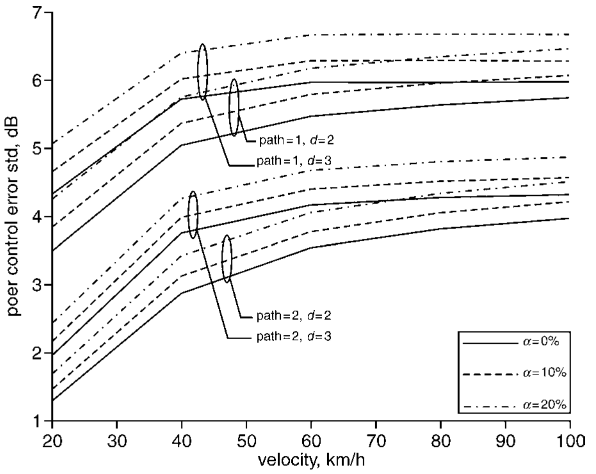

Fig. 3 *Analytical results of σ_z (dB) for $\alpha \neq 0$*

Conclusion: We have investigated CLPC performance for CDMA systems over Rayleigh fading channels, considering several non-ideal conditions such as the channel estimation error, PCB error, and multiple PCG delays. Our analytical approach can be used instead of the Monte-Carlo simulation, which typically requires a heavy computational load.

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References

1 Ariyavistakul, S., and Change, L.F.: 'Signal and interference statistics of a CDMA system with feedback power control', *IEEE Trans. Commun.*, 1993, **41**, (11), pp. 1626–1634
2 Song, L., Mandayam, N.B., and Gazic, Z.: 'Analysis of an up/down power control algorithm for the CDMA reverse link under fading', *IEEE J. Sel. Areas in Commun.*, 2001, **19**, (2), pp. 277–286
3 Jakes, W.C.: 'Microwave mobile communications' (Wiley, 1974)
4 Romero-Jerez, J.M., Tellez-Labao, A., and Diaz-Estrella, A.: 'Effect of power control imperfections on the reverse link of cellular CDMA networks under multipath fading', *IEEE Trans. Veh. Technol.*, 2001, **53**, (1), pp. 61–71