Non-Coherent and Mismatched-Coherent Receivers for Distributed STBCs with Amplify-and-Forward Relaying

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Abstract—Cooperative diversity is a transmission technique where multiple nodes in a network cooperate to form a virtual antenna array realizing the benefits of spatial diversity in a distributed fashion. The coherent scenario considered in most existing work on cooperative diversity assumes the availability of perfect channel state information at the relay and destination terminals and is highly unrealistic in practical applications. In this paper, we investigate non-coherent and mismatched-coherent receivers for a cooperative diversity scheme assuming both quasi-static and time-varying fading channels for the underlying cooperative links. Specifically, we consider a distributed space-time block coded (STBC) system in a single-relay scenario operating in the amplify-and-forward relaying mode. Exploiting the orthogonal structure of distributed STBC, we first derive a non-coherent decoding rule which can be implemented in practice by a Viterbi-type algorithm. Although this decoding rule has been derived assuming quasi-static channels, its inherent channel tracking capability allows its deployment over time-varying channels with a promising performance as a sub-optimal solution. As a possible alternative to non-coherent detection, we investigate the performance of mismatched-coherent receiver (i.e., coherent detection with imperfect channel estimation) within the considered relay-assisted transmission scenario. We further compare the performance of non-coherent and mismatched-coherent receivers to reveal their robustness under various mobility scenarios.

Index Terms—Cooperative diversity, space-time block coding, non-coherent detection, imperfect channel estimation, fading channels.

I. INTRODUCTION

THE revolutionary concept of space-time coding [1]–[4] introduced in the late 90's has demonstrated that the deployment of multiple antennas at the transmitter allows for increase in throughput and reliability. However, the use of antenna arrays is not practical for deployment in certain applications such as sensor networks due to the space limitations of small transceivers. This limitation motivates cooperation

between different nodes where a node attempts to use others' antennas to relay its message. Cooperative diversity, also known as user cooperation [5]–[10], exploits the broadcast nature of wireless transmission, creating a virtual antenna array through cooperating nodes. The pioneering works on cooperative diversity address information-theoretical aspects of cooperative networks investigating achievable rate regions and outage probabilities. The outage analysis in [7] relies on a random coding argument and demonstrates that full spatial diversity can be achieved using a rich set of codes. Laneman et. al. [7] suggest conventional orthogonal STBC, i.e., originally proposed for coding across co-located antennas in [3], for practical implementation of user cooperation in a distributed fashion. In [9], Nabar et. al. analyze distributed STBC operating in amplify-and-forward mode through the derivation of pairwise error probability (PEP) expression. They demonstrate that the original design criteria for conventional STBC (i.e., rank and determinant criteria) still apply for the design of distributed STBC schemes under the assumption that appropriate power control rules are employed at relay nodes.

The coherent scenario considered in [9] as well as the majority of the current works on cooperative diversity assume the availability of perfect channel state information (CSI) at the receiver. In practice, the fading channel coefficients are first estimated and then used in the detection process at the destination terminal. The quality of channel estimates affects the overall performance of cooperative transmission and might become a performance limiting factor. To the best of our knowledge, the very first papers addressing the channel estimation problem in the context of cooperative diversity are [11] and [12] which consider AF (amplify-and-forward) and DF (decode-and-forward) relaying modes, respectively. In [11], we have proposed a non-coherent decoder based on the so-called generalized maximum likelihood estimator for distributed STBC in AF mode. In [12], Chen and Laneman have proposed a non-coherent demodulator with a piecewise-linear combiner that accurately approximates the ML detector for cooperative diversity schemes with BFSK (binary frequency shift keying) modulation. In [13], Tarasak et. al. have developed a differential modulation scheme for a two-user cooperative diversity system which avoids channel estimation. Several non-coherent distributed STBCs have been further proposed in [14], [15], [16].

The works in [12]–[16] build upon the assumption of DF relaying and focus on only quasi-static fading channels. In

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contrary, our earlier work in [11] investigates the problem of non-coherent detection for AF relaying over time-varying channels. Built upon our promising results in [11], the current paper aims to provide a comprehensive treatment of non-coherent detection and coherent detection with imperfect channel estimation for distributed STBC schemes with AF relaying. Considering the cooperation protocol developed in [9], we first investigate ML detection for distributed STBCs without assuming channel knowledge. The form of the ML rule for time-varying channels does not lend itself to a practical implementation other than exhaustive search over all possible sequences. However, under the assumptions of quasi-static fading channel and the inherent orthogonality structure of STBCs, the loglikelihood function reduces to a simple form which can be implemented in practice by a Viterbi-type algorithm. Due to its inherent channel tracking ability, we further demonstrate that the derived decoding rule can be deployed over time-varying channels as a low-complexity sub-optimal solution. As a possible alternative to non-coherent detection, we investigate the performance of coherent detection with imperfect channel estimation¹ where the channel estimates are obtained through pilot symbols. It is further demonstrated through PEP derivations that both the proposed non-coherent and mismatched-coherent receivers are able to collect full diversity for quasi-static channels.

The rest of the paper is organized as follows: In Section II, we present the distributed STBC transmission model with AF relaying. In Section III, we derive non-coherent decoders for distributed STBCs over quasi-static and time-varying fading channels. In Section IV, we investigate pilot-assisted channel estimation within the considered relay-assisted transmission scenario and present the mismatched-coherent detector. In Section V, we present an extensive Monte Carlo simulation study to demonstrate a comparative performance analysis of considered non-coherent and mismatched-coherent receivers under various channel assumptions. Finally, we conclude in Section VI. The appendixes include derivations of PEP expressions for the diversity gain analysis of detection techniques under consideration.

Notation: $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ denote conjugate, transpose, and Hermitian transpose operations, respectively. $E[\cdot]$ denotes expectation, $\text{diag}(\cdot)$ stands for a diagonal matrix, $|\cdot|$ denotes the absolute value, $\|\cdot\|$ denotes the Euclidean norm of a vector, $\det(\cdot)$ denotes the determinant of a matrix, and $\mathbf{I}_N$ denotes the identity matrix of size N . Bold upper-case letters denote matrices and bold lower-case letters denote vectors.

II. TRANSMISSION MODEL

A wireless communication system scenario where the source terminal S transmits information to the destination terminal D with the assistance of relay terminal R is considered (See Fig. 1). As the user cooperation protocol, we adopt the so-called Protocol I proposed by Nabar et. al. [9]: Specifically, the source terminal communicates with the relay and destination terminals during the first signaling interval.

¹In the literature, this is sometimes referred as *mismatched* receiver [17]. In this paper, we refer it as "mismatched-coherent" detection to distinguish it from non-coherent detection.

Fig. 1. Schematic representation of relay-assisted transmission.

In the second signaling interval, both the relay and source terminals communicate with the destination terminal. For the relay-to-destination link, AF relaying is used, in which the relay terminal amplifies and re-transmits the signal received from the source terminal in the first signaling interval. Source and relay terminals are equipped with single transmit and receive antennas. Perfect synchronization is assumed among the cooperating terminals. Any linear modulation technique such as QAM (quadrature amplitude modulation) or PSK (phase shift keying) modulation can be used.

Let two consecutive signals transmitted by the source terminal be denoted as x_1 and x_2 . We assume M-PSK modulation with normalized unit energy for the signals, i.e., $E[|x_i|^2] = 1$, $i = 1, 2$. In the first signaling interval, the signal received at the relay terminal is given as

$$r_R = \sqrt{E_{SR}}h_{SR}x_1 + n_R \quad (1)$$

where E_{SR} represents the average energy available at the relay terminal considering the path loss and possible shadowing effects in source-to-relay $S \rightarrow R$ link. Here, h_{SR} denotes the complex fading coefficient over $S \rightarrow R$ link. It is modeled as a complex Gaussian random variable with variance 0.5 per dimension, which leads to a Rayleigh fading channel. n_R is a zero-mean complex Gaussian random variable with variance $N_0/2$ per dimension, which models the additive noise term. The signal received at the destination terminal in the first time slot is given by

$$r_{D,1} = \sqrt{E_{SD}}h_{SD}x_1 + n_{D,1} \quad (2)$$

where E_{SD} represents the average energy available at the destination terminal taking into account path loss and shadowing effects in source-to-destination $S \rightarrow D$ link. h_{SD} denotes the complex fading coefficient over $S \rightarrow D$ link modeled as a complex Gaussian random variable with variance 0.5 per dimension and $n_{D,1}$ is the additive noise term modeled as a zero-mean complex Gaussian random variable with variance $N_0/2$ per dimension. The relay terminal normalizes the received signal r_R by a factor of $\sqrt{E[|r_R|^2]}$ to ensure the unity of average energy and re-transmits the signal during the second time slot. Therefore, the received signal at the destination terminal in the second time slot is given as

$$r_{D,2} = \sqrt{E_{RD}}h_{RD}\frac{r_R}{\sqrt{E[|r_R|^2]}} + \sqrt{E_{SD}}h_{SD}x_2 + n_{D,2} \quad (3)$$

where $n_{D,2}$ is the additive noise term modeled as a zero-mean complex Gaussian random variable with variance $N_0/2$ per dimension. In (3), E_{RD} represents the average energy

$$\mathbf{r}_m \stackrel{\text{def}}{=} \begin{bmatrix} r_k \\ r_{k+1} \\ r_{k+2} \\ r_{k+3} \end{bmatrix} = \begin{bmatrix} \sqrt{E_{SD}}h_{2,k}x_{1,m} + n_k \\ \sqrt{\gamma_{1,k+1}}\sqrt{E_{RD}}h_{1,k+1}x_{1,m} + \sqrt{\gamma_{2,k+1}}\sqrt{E_{SD}}h_{2,k+1}x_{2,m} + n_{k+1} \\ -\sqrt{E_{SD}}h_{2,k+2}x_{2,m}^* + n_{k+2} \\ -\sqrt{\gamma_{1,k+3}}\sqrt{E_{RD}}h_{1,k+3}x_{2,m}^* + \sqrt{\gamma_{2,k+3}}\sqrt{E_{SD}}h_{2,k+3}x_{1,m}^* + n_{k+3} \end{bmatrix}, m = 1, 2, \dots, N/4, k = 4(m-1) \quad (10)$$

$$\mathbf{X}_m = \begin{bmatrix} 0 & \sqrt{E_{SD}}x_{1,m} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\gamma_{1,k+1}}\sqrt{E_{RD}}x_{1,m} & \sqrt{\gamma_{2,k+1}}\sqrt{E_{SD}}x_{2,m} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\sqrt{E_{SD}}x_{2,m}^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\sqrt{\gamma_{1,k+3}}\sqrt{E_{RD}}x_{2,m}^* & \sqrt{\gamma_{2,k+3}}\sqrt{E_{SD}}x_{1,m}^* \end{bmatrix} \quad (11)$$

available at the destination terminal considering the path loss and shadowing effects in relay-to-destination $R \rightarrow D$ link. h_{RD} denotes the complex fading coefficient over the same link and is modeled as a complex Gaussian random variable with variance 0.5 per dimension. Replacing the normalization factor $E[|r_R|^2] = E_{SR} + N_0$ and (1) in (3), we obtain

$$r_{D,2} = \sqrt{\frac{E_{RD}E_{SR}}{E_{SR} + N_0}}h_{RD}h_{SR}x_1 + \sqrt{E_{SD}}h_{SD}x_2 + \tilde{n} \quad (4)$$

with

$$\tilde{n} = \sqrt{\frac{E_{RD}}{E_{SR} + N_0}}h_{RD}n_R + n_{D,2}. \quad (5)$$

In (5), $\tilde{n}$ (conditioned on h_{RD}) is zero-mean complex Gaussian with variance of

$$E[|\tilde{n}|^2 | |h_{RD}|] = N_0 \left(1 + \frac{E_{RD}|h_{RD}|^2}{E_{SR} + N_0} \right). \quad (6)$$

We can rewrite the received signal normalizing (4) with (6) as

$$r_{D,2} = \sqrt{\gamma_1}\sqrt{E_{RD}}h_{RD}h_{SR}x_1 + \sqrt{\gamma_2}\sqrt{E_{SD}}h_{SD}x_2 + n \quad (7)$$

where n turns out to be zero-mean complex Gaussian random variable with variance $N_0/2$ per dimension. This normalization does not affect the signal-to-noise-ratio (SNR), but simplifies the ensuing presentation [9]. In (7), γ_1 and γ_2 are defined respectively, as

$$\gamma_1 = \frac{E_{SR}/N_0}{1 + E_{SR}/N_0 + |h_{RD}|^2 E_{RD}/N_0}, \quad (8)$$

$$\gamma_2 = \frac{1 + E_{SR}/N_0}{1 + E_{SR}/N_0 + |h_{RD}|^2 E_{RD}/N_0}. \quad (9)$$

After setting up the basic relay-assisted transmission model given by (2) and (7), we now introduce space-time coding across the transmitted signals x_1 and x_2 . Although different classes of space-time coding proposed originally for co-located antennas can be applied to cooperative diversity schemes in a distributed fashion, we employ STBC to exploit its inherent orthogonality; an essential feature for the proposed detection algorithms in the next section. For the case of single relay deployment (as considered here)², we use STBC designed for two transmit antennas, i.e., Alamouti's scheme

²Extensions to other orthogonal STBCs are straightforward, but not pursued here due to space limitations.

[4]. Considering the broadcasting and relaying phases, we need four time slots for the transmission of two Alamouti-coded symbols. Assume that the destination terminal makes an observation for a duration length of N (N is an even number). The received signal vector over four time slots is given by (10) which can be found on top of the page. In (10), $h_{1,k}$ and $h_{2,k}$ are defined as $h_{1,k} = h_{SR,k-1}h_{RD,k}$ and $h_{2,k} = h_{SD,k}$, respectively. Further introducing $\mathbf{h}_m = [h_{1,k}, h_{2,k}, \dots, h_{1,k+3}, h_{2,k+3}]^T$, $\mathbf{n}_m = [n_k, \dots, n_{k+3}]^T$, and $\mathbf{X}_m$ (c.f. (11)), the received signal vector over the whole observation period can be obtained as

$$\underbrace{\begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \dots \\ \mathbf{r}_{N/4} \end{bmatrix}}_{\mathbf{r}} = \underbrace{\begin{bmatrix} \mathbf{X}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{X}_{N/4} \end{bmatrix}}_{\mathbf{X}} \underbrace{\begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \dots \\ \mathbf{h}_{N/4} \end{bmatrix}}_{\mathbf{h}} + \underbrace{\begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \\ \dots \\ \mathbf{n}_{N/4} \end{bmatrix}}_{\mathbf{n}} \quad (12)$$

where $\mathbf{X}$ represents the data matrix with size $N \times 2N$, $\mathbf{h}$ is the channel vector with size $2N \times 1$, and $\mathbf{n}$ denotes the noise vector with size $N \times 1$.

III. NON-COHERENT DETECTION FOR DISTRIBUTED STBCs

In this section, we investigate non-coherent detection for distributed STBC systems without assuming channel knowledge over both time-varying and quasi-static fading channels.

A. Non-Coherent Detection over Time-Varying Fading Channels

First, we consider the general case where all underlying links experience time-selectivity. Conditioned on both the data matrix $\mathbf{X}$ and $R \rightarrow D$ link fading coefficients, the probability density function (pdf) of $\mathbf{r}$ can be written as [18]

$$p(\mathbf{r} | \mathbf{X}, h_{RD,l}, l = 0, 1, \dots, N-1) = (\pi^N \det(\mathbf{R}_{\mathbf{r}}))^{-1} \times \exp(-\mathbf{r}^H \mathbf{R}_{\mathbf{r}}^{-1} \mathbf{r}) \quad (13)$$

$$\mu(\mathbf{X}) = \arg \max_{\mathbf{X}} \left\{ \int_0^\infty \dots \int_0^\infty (\pi^N \det(\mathbf{R}_r))^{-1} \exp(-\mathbf{r}^H \mathbf{R}_r^{-1} \mathbf{r}) \times f(|\mathbf{h}_{RD}|) d|\mathbf{h}_{RD}| \right\} \quad (15)$$

$$\mathbf{X}_m = \begin{bmatrix} 0 & \sqrt{\gamma_1 E_{RD}} x_{1,m} & 0 & -\sqrt{\gamma_1 E_{RD}} x_{2,m}^* \\ \sqrt{E_{SD}} x_{1,m} & \sqrt{\gamma_2 E_{SD}} x_{2,m} & -\sqrt{E_{SD}} x_{2,m}^* & \sqrt{\gamma_2 E_{SD}} x_{1,m}^* \end{bmatrix}^T, m = 1, 2, \dots, N/4 \quad (17)$$

$$\mu(\mathbf{X}) = \underset{|\mathbf{h}_{RD}|}{E} \left[\arg \min_{\mathbf{X}} \left\{ N \ln(\pi) + \ln(\det(\mathbf{X} \mathbf{R}_h \mathbf{X}^H + N_0 \mathbf{I}_N)) + \mathbf{r}^H (\mathbf{X} \mathbf{R}_h \mathbf{X}^H + N_0 \mathbf{I}_N)^{-1} \mathbf{r} \right\} \right] \quad (21)$$

where $\mathbf{R}_r$ is defined as the autocorrelation matrix of $\mathbf{r}$ (conditioned on $\mathbf{X}$ and $h_{RD,i}$) and is given by

$$\mathbf{R}_r = \mathbf{X} \mathbf{R}_h \mathbf{X}^H + N_0 \mathbf{I}_N \quad (14)$$

with $\mathbf{R}_h$ denoting the autocorrelation matrix of $\mathbf{h}$ (conditioned on h_{RD}). Based on (13), it can be easily shown that the ML decision metric is given by (15). In (15), $f(|\mathbf{h}_{RD}|)$ is the joint density function of the fading coefficients' magnitudes over $R \rightarrow D$ link for the observation length of N . The direct implementation of (15) requires an exhaustive search over all possible sequences and computation of multidimensional integration, which make the practical decoder design infeasible especially for large sequence lengths. For the special case of non-fading $R \rightarrow D$ link, i.e., $\mathbf{h}_{RD} = [1, 1, \dots, 1]$, (15) assumes the form of

$$\mu(\mathbf{X}) = \arg \min_{\mathbf{X}} \left\{ \ln(\det(\mathbf{R}_r)) + \mathbf{r}^H \mathbf{R}_r^{-1} \mathbf{r} \right\}. \quad (16)$$

Non-fading assumption can be justified in practical scenarios where the destination and relay terminals have a strong line-of-sight connection [9]. The resulting form of ML metric for such a scenario avoids the integrals due to the non-fading nature of $R \rightarrow D$ link, however still requires an exhaustive search over all possible sequences. Since the argument of the determinant term in (16) is dependent on data sequence, it is not possible to derive a recursive expression directly from this rule. However, an indirect approach, similar to [18], is possible to derive for the considered case by first filtering the received sequence to transform the determinant term into a diagonal one, which will then lend itself to the derivation of a recursive form. This requires computation of prediction filter coefficients for branch metrics at each step bringing additional complexity and will not be pursued here.

B. Non-Coherent Detection over Quasi-Static Fading Channels

Now we return our attention to the quasi-static fading case where the fading coefficients of all underlying links are assumed to remain constant over the observation period. For this case, $\mathbf{X}_m$ reduces to (17). The received signal vector can

be now written in the matrix form as

$$\underbrace{\begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \dots \\ \mathbf{r}_{N/4} \end{bmatrix}}_{\mathbf{r}} = \underbrace{\begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_{N/4} \end{bmatrix}}_{\mathbf{X}} \underbrace{\begin{bmatrix} h_1 \\ h_2 \end{bmatrix}}_{\mathbf{h}} + \underbrace{\begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \\ \dots \\ \mathbf{n}_{N/4} \end{bmatrix}}_{\mathbf{n}} \quad (18)$$

where $h_1 = h_{SR}h_{RD}$, $h_2 = h_{SD}$, $\mathbf{X}$ is the data matrix with size $N \times 2$, and $\mathbf{n}$ denotes a $N \times 1$ noise vector. Here, we have dropped the subscripts denoting time in the fading coefficients' representation since they are already assumed to be constant over N intervals. Under the quasi-static channel assumption, the ML decision metric is obtained as

$$\mu(\mathbf{X}) = \arg \max_{\mathbf{X}} \left\{ \underset{|\mathbf{h}_{RD}|}{E} \left[\underbrace{\left(\pi^N \det(\mathbf{R}_r) \right)^{-1} \exp(-\mathbf{r}^H \mathbf{R}_r^{-1} \mathbf{r})}_{p(\mathbf{r}|\mathbf{X}, h_{RD})} \right] \right\} \\ = \arg \max_{\mathbf{X}} \left\{ \int_0^\infty p(\mathbf{r}|\mathbf{X}, h_{RD}) f(|h_{RD}|) d|h_{RD}| \right\}. \quad (19)$$

The integral of (19) with respect to h_{RD} does not readily yield a closed-form solution. As an alternative solution, we modify the decision metric as³

$$\mu(\mathbf{X}) = \underset{|\mathbf{h}_{RD}|}{E} \left[\arg \max_{\mathbf{X}} \left\{ \left(\pi^N \det(\mathbf{R}_r) \right)^{-1} \exp(-\mathbf{r}^H \mathbf{R}_r^{-1} \mathbf{r}) \right\} \right] \quad (20)$$

Replacing (14) in (20), we obtain (21). Using the matrix identity

$(\mathbf{A} + \mathbf{BCD})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1} \mathbf{B} (\mathbf{C}^{-1} + \mathbf{DA}^{-1} \mathbf{B})^{-1} \mathbf{DA}^{-1}$ [19], the argument of the determinant term in (21) can be rearranged as

$$(\mathbf{X} \mathbf{R}_h \mathbf{X}^H + N_0 \mathbf{I}_N)^{-1} = \frac{\mathbf{I}_N}{N_0} - \frac{1}{N_0^2} \mathbf{X} \left(\mathbf{R}_h^{-1} + \frac{1}{N_0} \mathbf{X}^H \mathbf{X} \right)^{-1} \mathbf{X}^H \quad (22)$$

Further replacing $\mathbf{R}_h = \text{diag}(|h_{RD}|^2, 1)$ in (22) and exploiting the embedded orthogonality of data sequence in the resulting expression, i.e.,

³Although the resulting decision rule is not optimal in ML sense, our observations through Monte-Carlo simulation indicate that (20) presents nearly identical performance to that achievable through the optimal metric given by (19).

$\mathbf{X}^H \mathbf{X} = \text{diag}((N/2)\gamma_1 E_{RD}, (N/2)(\gamma_2 + 1)E_{SD})$, we rewrite (22) as

$$(\mathbf{X}\mathbf{R}_h\mathbf{X}^H + N_0\mathbf{I}_N)^{-1} = \frac{\mathbf{I}_N}{N_0} - \frac{1}{N_0^2}\mathbf{X}\mathbf{A}\mathbf{X}^H \quad (23)$$

where $\mathbf{A}$ is defined by

$$\mathbf{A} = \begin{bmatrix} \frac{|h_{RD}|^2}{1 + \frac{N}{2}\gamma_1 \frac{E_{RD}}{N_0}|h_{RD}|^2} & 0 \\ 0 & \frac{1}{1 + \frac{N}{2}(\gamma_2 + 1) \frac{E_{SD}}{N_0}} \end{bmatrix}$$

Using the matrix identity $\det(\mathbf{I} + \mathbf{A}\mathbf{B}) = \det(\mathbf{I} + \mathbf{B}\mathbf{A})$ [19] and again exploiting the orthogonality structure of the underlying STBC, the determinant term in (21) can be shown to be independent of the data sequence. Replacing (23) in (21) and dropping unnecessary terms which do not affect maximization, we obtain

$$\mu(\mathbf{X}) = \mathbb{E}_{|h_{RD}|} \left[\arg \min_{\mathbf{X}} \left\{ -\frac{1}{N_0^2} \mathbf{r}^H \mathbf{X} \mathbf{A} \mathbf{X}^H \mathbf{r} \right\} \right]. \quad (24)$$

The expectation of (24) with respect to h_{RD} does not yield a closed-form solution for the general case. In the following, we demonstrate that a closed-form decision metric can be obtained imposing some assumptions on the underlying links.

Scenario 1 (High SNR in link and perfect power control between $S \rightarrow D$ and $R \rightarrow D$ links): Assume perfect power control where $S \rightarrow D$ and $R \rightarrow D$ links are balanced, i.e., $E_{RD}/N_0 = E_{SD}/N_0$, and sufficiently large SNR for the $S \rightarrow D$ link, i.e., $E_{SR}/N_0 \gg E_{SD}/N_0$. Under these assumptions, $\mathbf{A}$ reduces to the form of

$$\mathbf{A} = \begin{bmatrix} \frac{|h_{RD}|^2}{1 + (N/2)(E_{SD}/N_0)|h_{RD}|^2} & 0 \\ 0 & \frac{1}{1 + N(E_{SD}/N_0)} \end{bmatrix}.$$

Replacing $\mathbf{A}$ in (24) and carrying out the expectation with respect h_{RD} , we obtain the decision metric as

$$\mu(\mathbf{X}) = \arg \max_{\mathbf{X}} \left\| \Phi \mathbf{X}^H \mathbf{r} \right\|^2 \quad (25)$$

where

$$\Phi = \begin{bmatrix} \sqrt{\beta(1 - \beta e^{\beta} \Gamma(0, \beta))} & 0 \\ 0 & \sqrt{\frac{1}{1 + N(E_{SD}/N_0)}} \end{bmatrix} \quad (26)$$

with $\beta = 2/(NE_{SD}/N_0)$ and $\Gamma(\cdot, \cdot)$ denotes the incomplete gamma function [20]. Under the assumption of $E_{RD}/N = E_{SD}/N_0 \gg 1$, Φ simplifies to

$$\Phi = \begin{bmatrix} \sqrt{\frac{2}{N(E_{SD}/N_0)}} & 0 \\ 0 & \sqrt{\frac{1}{N(E_{SD}/N_0)}} \end{bmatrix} \quad (27)$$

which lets us to avoid the use of incomplete gamma function in the decision metric.

Scenario 2 (Non-fading $R \rightarrow D$ link): Assume $R \rightarrow D$ link is non-fading, i.e., $h_{RD} = 1$, perfect power control where $S \rightarrow D$ and $R \rightarrow D$ links are balanced, i.e., $E_{RD}/N_0 = E_{SD}/N_0$, and sufficiently large SNR for the $S \rightarrow R$ link, i.e., $E_{SR}/N_0 \gg E_{SD}/N_0$. It can be easily

shown that the decision metric is still given by (25) where Φ is

$$\Phi = \begin{bmatrix} \sqrt{\frac{1}{1 + (N/2)(E_{SD}/N_0)}} & 0 \\ 0 & \sqrt{\frac{1}{1 + N(E_{SD}/N_0)}} \end{bmatrix} \quad (28)$$

One can interestingly note that (25) has a similar form to

$$\mu(\mathbf{X}) = \arg \max_{\mathbf{X}} \left\| \mathbf{X}^H \mathbf{r} \right\|^2 \quad (29)$$

which is the quadratic non-coherent receiver derived earlier for conventional (i.e., non-distributed) STBC in [21], [22]. Replacing Φ in (25) by an identity matrix, one can simply obtain (29). In other words, (25) is a generalized version of the conventional decoder. In the considered cooperative scenario, the effects of AF relaying and path loss/shadowing associated with underlying relay links manifest themselves in (25) through the scaling matrix Φ . Through the derivation of PEP in the Appendix I, we demonstrate this the non-coherent decoding rule is able to collect a diversity order of two which is the full diversity for the considered single-relay scenario. It should be noted that this diversity order is attainable for asymptotically high SNR. In practical range of SNR values, the performance of distributed STBC experiences some degradation in comparison to that of non-distributed STBC. In the practical implementation of (25), one should notice that the quadratic nature of (25) results in a phase ambiguity. This ambiguity can be resolved easily by fixing the first symbol transmitted from each of the links to a specific value, i.e., insertion of a known symbol at the beginning of the frame [21]. Direct implementation of (25) is still impractical due to substantial computational complexity. However, this time, unlike (15) and (16), it is possible to develop a recursive expression from (25) which can be easily implemented by a Viterbi-type algorithm. Expanding (25), we can readily obtain the desired recursive form given by (30). The inner term in (30) can be interpreted as a channel estimate based on a truncation interval of previous J sub-blocks. Although the derived decoding rule is based on a quasi-static fading assumption, this inherent channel tracking ability makes it a suitable candidate for deployment over time-varying channels

IV. MISMATCHED-COHERENT DETECTION FOR DISTRIBUTED STBCs

In this section, we consider a mismatched-coherent receiver, i.e., coherent detection with imperfect channel estimation, as a possible alternative to the non-coherent receiver proposed in the previous section. For channel estimation purposes, we employ pilot symbols (i.e., a set of symbols whose location and values are known to the receiver) multiplexed with the information-bearing data. To the best of our knowledge, pilot-symbol-assisted channel estimation has not been yet fully studied in the context of cooperative communication.

A. Mismatched-Coherent Detection over Quasi-Static Fading Channels

Let $\mathbf{X}_T$ denote the pilot symbol matrix transmitted by the source terminal followed by a data matrix $\mathbf{X}_D$. Under the

assumption that fading coefficients are constant over the entire transmission frame, the received signals during the training and data transmission phases are given, respectively, as

$$\mathbf{r}_T = \mathbf{X}_T \mathbf{h} + \mathbf{n}_T, \quad (31)$$

$$\mathbf{r}_D = \mathbf{X}_D \mathbf{h} + \mathbf{n}_D, \quad (32)$$

where $\mathbf{h} = (h_1, h_2)^T$ with $h_1 = h_{SR}h_{RD}$ and $h_2 = h_{SD}$. In (31)-(32), the entries of $\mathbf{n}_T$ and $\mathbf{n}_D$ are zero-mean complex Gaussian random variables with variance $N_0/2$ per dimension. The channel coefficients are first estimated using $\mathbf{r}_T$ and $\mathbf{X}_T$ to obtain the channel estimate $\hat{\mathbf{h}}$, which can be then used to minimize the following coherent ML metric

$$\mu(\mathbf{X}_D) = \arg \min_{\mathbf{X}_D} \left\| \mathbf{r}_D - \mathbf{X}_D \hat{\mathbf{h}} \right\|^2 \quad (33)$$

as if the channel was perfectly known. The performance of the mismatched receiver relies on the choice of the channel estimator. Here, we consider two training-based channel estimation techniques, namely, ML and linear-minimum-mean-square-error (LMMSE) estimators.

ML Estimator: Conditioned on $\mathbf{h}$ and $\mathbf{X}_T$, the ML estimate of the channel matrix is amount to minimizing $\|\mathbf{r}_T - \mathbf{X}_T \mathbf{h}\|^2$ with respect to $\mathbf{h}$. Therefore, under the similar SNR assumptions imposed in the previous section (i.e., $E_{RD}/N_0 = E_{SD}/N_0$, $E_{SR}/N_0 \gg E_{SD}/N_0$), the ML estimate of $\mathbf{h}$ is given as [23]

$$\hat{\mathbf{h}}_{ML} = (\mathbf{X}_T^H \mathbf{X}_T)^{-1} \mathbf{X}_T^H \mathbf{r}_T \quad (34)$$

Replacing (31) in (34), we obtain

$$\hat{\mathbf{h}}_{ML} = \mathbf{h} + \varepsilon_{ML} \quad (35)$$

where we define the channel estimation error vector $\varepsilon_{ML} = (\mathbf{X}_T^H \mathbf{X}_T)^{-1} \mathbf{X}_T^H \mathbf{n}_T$ with covariance matrix

$$\mathbf{E}[\varepsilon_{ML} \varepsilon_{ML}^H] = \begin{bmatrix} \frac{1}{(P/2)(E_{SD}/N_0)} & 0 \\ 0 & \frac{1}{P(E_{SD}/N_0)} \end{bmatrix} \quad (36)$$

where P denotes the number pilot symbols.

LMMSE estimator: The LMMSE estimate of the channel matrix can be obtained as $\hat{\mathbf{h}}_{LMMSE} = \mathbf{B} \mathbf{r}_T$, where $\mathbf{B}$ is a 2×2 matrix obtained through minimizing $E[\|\mathbf{B} \mathbf{r}_T - \mathbf{h}\|^2]$ with respect to $\mathbf{B}$. Using the result given in [24], the channel estimate $\hat{\mathbf{h}}_{LMMSE}$ is obtained as

$$\hat{\mathbf{h}}_{LMMSE} = (\mathbf{X}_T^H \mathbf{X}_T + N_0 \Delta_{\mathbf{h}}^{-1})^{-1} \mathbf{X}_T^H \mathbf{r}_T \quad (37)$$

where $\Delta_{\mathbf{h}} = \mathbf{I}_2$ is the autocorrelation matrix of $\mathbf{h}$. Replacing (31) in (37), we obtain

$$\hat{\mathbf{h}}_{LMMSE} = (\mathbf{X}_T^H \mathbf{X}_T + N_0 \Delta_{\mathbf{h}}^{-1})^{-1} \mathbf{X}_T^H \mathbf{X}_T \mathbf{h} + \varepsilon_{LMMSE} \quad (38)$$

where we define the channel estimation error vector $\varepsilon_{LMMSE} = (\mathbf{X}_T^H \mathbf{X}_T + N_0 \Delta_{\mathbf{h}}^{-1})^{-1} \mathbf{X}_T^H \mathbf{n}_T$. Under the assumption of $E_{SD}/N_0 \rightarrow \infty$, the covariance matrix of ε_{LMMSE} converges to (36). In Appendix II, we demonstrate through PEP derivation that the mismatched-coherent receiver is able to achieve full diversity over quasi-static channels.

Fig. 2. Frame structure for pilot-symbol-assisted channel estimation.

B. Mismatched-Coherent Detection over Time-Varying Fading Channels

In this subsection, we assume all underlying links experience time-selectivity. In the considered single-relay scenario, Alamouti-coded pilot symbols are sent over four time slots: Specifically, in time slot k , the source terminal simultaneously transmits the pilot symbol p_1 to the destination terminal D and the relay terminal R . In time slot $k+1$, the relay terminal transmits the received version of pilot symbol (after energy normalization) to the destination terminal D . In the same time slot, the source terminal transmits the other pilot symbol p_2 to the destination terminal. The same procedure is repeated for the transmission of $-p_2^*$ and p_1^* . The destination terminal makes an observation of L frames, each of which consists of N symbols and extracts $4L$ observations of the nearest pilot symbols for channel estimation purpose (Fig. 2). The destination terminal interpolates these observations (using a Wiener filter) to form an estimate of the channel coefficient at the data symbol to be detected [25]. Assume that the pilot symbols are located at the first 4 time slots of each frame, and let the received signal vector representing the observed pilot symbols denoted by $\bar{\mathbf{r}}_l = [\mathbf{r}_{l, -\lfloor (L-1)/2 \rfloor} \cdots \mathbf{r}_{l, \lfloor L/2 \rfloor}]$ where $\mathbf{r}_{l,k} = [r_{l+k}^0 \ r_{l+k}^1 \ r_{l+k}^2 \ r_{l+k}^3]$ for $k = \lfloor -(L-1)/2 \rfloor, \dots, 0, \dots, \lfloor L/2 \rfloor$. The fading estimates for $S \rightarrow D$ and $S \rightarrow R \rightarrow D$ links at the n^{th} symbol period in the l^{th} frame are then obtained as

$$\hat{h}_{1,l}^n = \mathbf{E}[h_{SD,l}^n \bar{\mathbf{r}}_l^H] \text{Cov}(\bar{\mathbf{r}}_l)^{-1} \bar{\mathbf{r}}_l \quad (39)$$

$$\hat{h}_{2,l}^n = \mathbf{E}[h_{RD,l}^n h_{SR,l}^n \bar{\mathbf{r}}_l^H] \text{Cov}(\bar{\mathbf{r}}_l)^{-1} \bar{\mathbf{r}}_l \quad (40)$$

V. NUMERICAL RESULTS

In this section, we present an extensive Monte-Carlo simulation study to demonstrate the error rate performance of the non-coherent and mismatched-coherent distributed STBC schemes considered so far. In our simulation study, we consider BPSK modulation and assume $E_{SD} = E_{RD}$, i.e., $S \rightarrow D$ and $R \rightarrow D$ links are balanced, which can be achieved through power control. Unless otherwise noted, we assume $E_{SR}/N_0 = 35\text{dB}$ and deployment of one receive antenna at the destination terminal.

In Fig.3, we present the bit error rate (BER) performance of the ML-optimum decoding rule given by (19) and the derived decoding rule given by (25) in conjunction with scaling matrix Φ given by (26), (27), and (28). We assume quasi-static Rayleigh fading for all underlying links. It is observed from Fig.3 that optimum non-coherent decoding rule and proposed decoder yield nearly identical performance. This

Fig. 3. BER performance of the derived non-coherent ML decoders for distributed STBC over the quasi-static fading $S \rightarrow R$, $S \rightarrow D$ and $R \rightarrow D$ links.

Fig. 4. BER performance of the proposed non-coherent ML decoder given by Eq. (23) and its recursive implementation given by Eq. (28) over the quasi-static fading $S \rightarrow R$ and $S \rightarrow D$ links.

is very promising since the implementation complexity of (25) is much lower than the optimum decoder which requires multi-dimensional integral and does not yield a closed-form solution. Considering the nearly identical performance observed for three different scaling matrices, (27) and (28) are favorable choices since they avoid the use of incomplete gamma function unlike (26). As a benchmark, we also investigate the performance of the non-coherent decoder rule given by (29) for conventional (i.e., non-distributed) STBC within the considered cooperative scenario. A performance loss of 1dB at $\text{BER} = 10^{-3}$ is observed. This performance degradation decreases further with the increasing number of receive antennas employed at the destination terminal. It should be emphasized (29) does not require any kind of information on the underlying links unlike (26)-(27)-(28) which need average SNR estimation for the underlying links. Therefore, (29) can be possibly used as an alternative sub-optimal detector for distributed STBC.

Fig. 5. Effective diversity order (EDO) of distributed STBC.

In Fig. 4, we provide further results on the performance of the proposed decoding rule (25) to discuss the effect of fading in the $R \rightarrow D$ link. Under the assumption of high $E_{SR}/N_0 = 35\text{dB}$ values, it is observed that a diversity order of two is achieved confirming our diversity gain analysis through PEP derivation in the Appendix I. The presence of fading in $R \rightarrow D$ link results in some performance degradation, e.g., 1.2dB at $\text{BER} = 10^{-3}$. This performance degradation is essentially a coding gain loss, i.e., horizontal shift in the performance, but the asymptotical diversity order remains unaffected. This can be more clearly seen in Fig. 5 where we plot the *effective* diversity order gains, i.e., $\log \text{PEP} / \log (E_{SD}/N_0)$ [26] relying on the PEP expressions (48) and (49) obtained for fading and non-fading $R \rightarrow D$ links, respectively. It is observed that these curves converge to two in the asymptotical high SNR attaining the maximum achievable diversity order. However, the convergence of (48) is rather slow due to the presence of the term $\log (E_{SD}/N_0)$ reflecting the effect of fading in the $R \rightarrow D$ link. A diversity loss is observed for low E_{SR}/N_0 values, c.f. the plots obtained for $E_{SR}/N_0 = 0$. In Fig. 4, we also include the performance of recursive implementation of (25). This requires a Viterbi-type decoding algorithm with the metric given as (30). We assume a 4-state trellis, which corresponds to a truncation to the previous sub-block, i.e., $J=1$. We choose the frame length $N=8$, resulting in a pilot insertion rate (PIR) of $1/4$. The simulation results clearly illustrate that the recursive implementation of the proposed decoder demonstrates a close performance to that of the one given by (25). The performance degradation is about 1.3dB which can be further decreased by increasing the truncation length. In Fig. 6, we illustrate the performance of mismatched-coherent receiver for distributed STBC over quasi-static fading channels. As a benchmark, we include the performance of genie receiver which assumes perfect CSI. For mismatched-coherent receiver, we use the ML channel estimator as given by (34). The frame length is chosen as $N=64$. Our simulation results indicate that the mismatched receiver performance loss with respect to the genie receiver is $\approx 3\text{dB}$ at $\text{BER} = 10^{-3}$ with $P=1$, however diversity order

Fig. 6. Performance comparison of mismatched-coherent and non-coherent decoders for distributed STBC over quasi-static fading links.

Fig. 7. BER performance comparison of the non-coherent optimal decoder, non-coherent recursive decoder, and mismatched-coherent decoder for distributed STBC with non-fading $R \rightarrow D$ link. Normalized Doppler values for $S \rightarrow R$ and $S \rightarrow D$ links are $fT_{SR} = fT_{SD} = 0.01$.

remains unaffected confirming our diversity gain analysis in the Appendix II. As the number of pilot symbols increases, the performance degradation decreases, e.g., ≈ 1.2 dB with $P=6$.

In Fig. 6, we also include the performance of the non-coherent decoder for further comparison with mismatched-coherent receiver. Assuming $P=1^4$, the proposed non-coherent receiver given by (25) outperforms the mismatched-coherent receiver by ≈ 1.8 dB at $\text{BER}=10^{-3}$. The recursive implementation of the non-coherent receiver given by (30) still outperforms the mismatched-coherent receiver by ≈ 0.5 dB at $\text{BER}=10^{-3}$. With the increasing number of pilot symbols (therefore at the cost of decreasing throughput), mismatched-coherent receiver is able to provide a better performance in comparison to its competitor.

In Fig. 7, we compare the performance of non-coherent and mismatched-coherent receivers within a time-varying channel

⁴It should be noted that only one pilot symbol is required for the non-coherent decoder to resolve the phase ambiguity.

Fig. 8. BER performance comparison of the non-coherent recursive decoder and mismatched-coherent decoder for distributed STBC with fading $R \rightarrow D$ link. Normalized Doppler values for $S \rightarrow R$, $R \rightarrow D$ and $S \rightarrow D$ links are $fT = fT_{SR} = fT_{RD} = fT_{SD} = 0.01, 0.02, 0.03, 0.05$.

scenario. We set $\text{PIR}=1/4$ and assume that $S \rightarrow R$ and $S \rightarrow D$ fading links experience Bessel-type autocorrelation function with normalized Doppler values of $fT_{SR} = fT_{SD} = 0.01$. A non-fading channel is assumed for the $R \rightarrow D$ link. We observe that the recursive implementation of the proposed non-coherent receiver, i.e., (30), still demonstrates close results in comparison to the optimum decoder given by (17) for time-varying channels although it is originally derived under the quasi-static channel assumption. Our simulation results illustrate that both receivers provide similar performance in the lower E_{SD}/N_0 region. As E_{SD}/N_0 increases, the non-coherent decoder is able to outperform slightly the mismatched-coherent receiver.

In Fig. 8, we consider a similar scenario as in the above, but a time-varying channel model is also considered for the $R \rightarrow D$ link. We assume the normalized Doppler values experienced in $S \rightarrow R$, $S \rightarrow D$, and $R \rightarrow D$ links are identical, i.e., $fT = fT_{SR} = fT_{SD} = fT_{RD}$. We observe that, for $fT = 0.01$ and $fT = 0.02$, the non-coherent decoder is able to provide a better performance. For $fT = 0.05$, it is observed that that mismatched-coherent receiver performs better although both receivers suffer from an error floor for this specific Doppler value. From the comparison of Figs. 7 and 8, we further observe that the presence of time-selective fading in $R \rightarrow D$ link with results in a performance degradation of ≈ 3 dB at $\text{BER}=10^{-3}$ for both decoders.

To have further insight into the performance comparison of non-coherent and mismatched-coherent decoders under various mobility scenarios, we plot the BER performance vs. normalized Doppler values in Fig. 9. Here, we fix the E_{SD}/N_0 at 15 dB. It is observed that the non-coherent decoder is able to outperform the mismatched-coherent decoder for low and intermediate Doppler values. For higher Doppler values, it can be seen that the mismatched-coherent decoder outperforms its competitor. In Fig. 10, we further let the Doppler values in the underlying links be different from each other. Specifically, we assume the following three scenarios:

Fig. 9. BER vs. Doppler spread for distributed STBC with non-coherent recursive decoder and mismatched-coherent decoder.

Fig. 10. BER vs. Doppler spread for distributed STBC with non-coherent recursive decoder and mismatched-coherent decoder.

- S.1) $fT_{RD} = fT_{SD} = 0.01$, $fT_{SR} = fT = [0.01, 0.06]$.
- S.2) $fT_{SR} = fT_{SD} = 0.01$, $fT_{RD} = fT = [0.01, 0.06]$.
- S.3) $fT_{RD} = fT_{SR} = 0.01$, $fT_{SD} = fT = [0.01, 0.06]$.

Our simulation results reveal that the first two scenarios yield identical performance. This is due the symmetrical structure of $S \rightarrow R$ and $R \rightarrow D$ links in the cascaded relay channel. Thus, for the sake of presentation simplicity, we only include the performance of the first scenario in the figure. For scenarios in S.1) and S.3), we observe that the non-coherent decoder is able to outperform mismatched-coherent decoder for normalized Doppler values less than 0.045 and 0.015, respectively.

VI. CONCLUSION

In this paper, we have investigated non-coherent and mismatched-coherent receivers for a relay-assisted transmission scheme using STBC and operating in AF mode. First, we have studied ML sequence estimation for the distributed STBC scheme under consideration without assuming channel knowledge. Under quasi-static fading assumption, the ML rule

reduces to a simple form due to the orthogonality structure of STBC, which reminds the conventional correlation receiver in a matrix notation. The PEP analysis demonstrates that the decoder is able to achieve the maximum diversity order. It also allows us to develop a recursive expression, which can be implemented in practice with a Viterbi-type algorithm. We have further demonstrated through simulation that such a receiver can be also used efficiently over time-varying channels. As a competing scheme, we have further considered mismatched-coherent receiver within the relay-assisted scenario. Our PEP analysis demonstrates that the mismatched-coherent receiver is able to collect the full diversity as its non-coherent competitor over quasi-static channels. We have presented a comprehensive performance comparison between competing receivers assuming various Doppler spreads experienced in the underlying cooperative links. Our results indicate that the non-coherent receiver is able to outperform mismatched-coherent receiver for low Doppler spreads while the mismatched receiver becomes a better choice for relatively higher Doppler spreads.

APPENDIX I

In this Appendix, we present a diversity gain analysis through the PEP derivation for the non-coherent detection rule presented in Section III.B. The PEP represents the probability that the receiver decides incorrectly for the codeword matrix $\mathbf{X}_e$ when $\mathbf{X}_c$ is transmitted. Let the codeword transmitted be $\mathbf{X}_c = [\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_{N/4}]^T$. Conditioned on h_{RD} , the asymptotic PEP for non-coherent receivers is readily found as [27, Eq.28]

$$P(\mathbf{X}_c \rightarrow \mathbf{X}_e | h_{RD}) = \left(\prod_{m=1}^2 \lambda_m \right)^{-1} \sum_{k=0}^2 \binom{4-k}{2} \times \frac{1}{k!} \left(\ln \left(\frac{|\mathbf{R}_{cc}|}{|\mathbf{R}_{ee}|} \right) \right)^k, \quad (41)$$

where $\mathbf{R}_{cc} = \mathbf{X}_c^H \mathbf{X}_c$ and $\mathbf{R}_{ee} = \mathbf{X}_e^H \mathbf{X}_e$ denote the autocorrelation matrices of $\mathbf{X}_c$ and $\mathbf{X}_e$, respectively. In (41), λ_m , $m = 1, 2$ are the positive eigenvalues of the matrix $\Psi_{ce} = (\mathbf{R}_r / \mathbf{X}_c) (\kappa_e - \kappa_c)$ where $\mathbf{R}_r / \mathbf{X}_c = \mathbf{X}_c \Sigma \mathbf{X}_c^H + \mathbf{I}_N$, $\kappa_c = -\mathbf{X}_c (\mathbf{R}_{cc} + \Sigma^{-1})^{-1} \mathbf{X}_c^H$, and $\kappa_e = -\mathbf{X}_e (\mathbf{R}_{ee} + \Sigma^{-1})^{-1} \mathbf{X}_e^H$. Here, Σ is defined as

$$\Sigma = \begin{bmatrix} \gamma_1 (E_{RD}/N_0) |h_{RD}|^2 & 0 \\ 0 & \gamma_2 (E_{SD}/N_0) \end{bmatrix}. \quad (42)$$

Using the definitions of κ_c and κ_e , we can write Ψ_{ce} in the following form

$$\Psi_{ce} = \left(\mathbf{X}_e \Sigma \mathbf{X}_c^H + \mathbf{I}_N \right) \left(\mathbf{X}_c (\mathbf{R}_{cc} + \Sigma^{-1})^{-1} \mathbf{X}_c^H - \mathbf{X}_e (\mathbf{R}_{ee} + \Sigma^{-1})^{-1} \mathbf{X}_e^H \right), \quad (43)$$

where we define the cross-correlation matrices of $\mathbf{X}_e$ and $\mathbf{X}_c$ as $\mathbf{R}_{ce} = \mathbf{X}_c^H \mathbf{X}_e$ and $\mathbf{R}_{ec} = \mathbf{X}_e^H \mathbf{X}_c$. After some algebra and using the fact that the non-zero eigenvalues of $\mathbf{AB}$ and $\mathbf{BA}$ are equal [19], we can rewrite (43) as

$$\begin{aligned} \Psi_{ce} = & \Sigma \mathbf{R}_{cc} (\Sigma^{-1} + \mathbf{R}_{cc})^{-1} \mathbf{R}_{cc} - \Sigma \mathbf{R}_{ce} (\Sigma^{-1} + \mathbf{R}_{ee})^{-1} \mathbf{R}_{ee} \\ & + (\Sigma^{-1} + \mathbf{R}_{cc})^{-1} \mathbf{R}_{cc} - (\Sigma^{-1} + \mathbf{R}_{ee})^{-1} \mathbf{R}_{ee} \end{aligned} \quad (44)$$

Under the high SNR assumption, i.e., $E_{SR}/N_0 \gg 1$, $E_{SD}/N_0 \gg 1$ and $E_{SR}/N_0 \gg E_{RD}/N_0 \gg 1$, Σ^{-1} term can be ignored and (44) reduces to

$$\Psi_{ce} = \Sigma (\mathbf{R}_{cc} - \mathbf{R}_{ce} (\mathbf{R}_{ee})^{-1} \mathbf{R}_{ec}). \quad (45)$$

The eigenvalues of Ψ_{ce} are then obtained as $\lambda_1 = (E_{RD}/N_0) |h_{RD}|^2 \chi_1$ and $\lambda_2 = (E_{SD}/N_0) \chi_2$ where χ_i , $i = 1, 2$ denote the eigenvalues of $(\mathbf{R}_{cc} - \mathbf{R}_{ce} (\mathbf{R}_{ee})^{-1} \mathbf{R}_{ec})$. Replacing the eigenvalues of (45) in (41) and noting $\mathbf{R}_{cc} = \mathbf{R}_{ee}$ (due to the orthogonal structure), (41) simplifies to⁵

$$P(\mathbf{X}_{cc} \rightarrow \mathbf{X}_{ee}) \approx 6 \frac{E}{|h_{RD}|} \left(\left(\frac{E_{RD}}{N_0} \chi_1 |h_{RD}|^2 + \delta \right)^{-1} \right) \times \left(\frac{E_{SD}}{N_0} \chi_2 \right)^{-1}. \quad (46)$$

Under the assumption of perfect power control, i.e., $E_{SD} = E_{RD}$, the unconditional PEP for the non-coherent receiver is found as

$$P(\mathbf{X}_{cc} \rightarrow \mathbf{X}_{ee}) \approx 6 \left(\frac{E_{SD}}{N_0} \right)^{-2} \left(\prod_{i=1}^2 \chi_i \right)^{-1} e^{\delta \left(\frac{E_{SD}}{N_0} \chi_1 \right)^{-1}} \times \Gamma \left(0, \delta \left(\frac{E_{SD}}{N_0} \chi_1 \right)^{-1} \right) \quad (47)$$

where $\Gamma(\cdot, \cdot)$ denotes the incomplete gamma function [20]. Using the limiting expression $\Gamma(0, z) \approx -\log(z)$ for $z \rightarrow 0$ [20] and noting that $e^{\delta \left(\frac{E_{SD}}{N_0} \chi_1 \right)^{-1}} \approx 1$, we can write (47) as

$$P(\mathbf{X}_{cc} \rightarrow \mathbf{X}_{ee}) \approx 6 \left(\frac{E_{SD}}{N_0} \right)^{-2} \log \left(\frac{E_{SD}}{N_0} \right) \left(\prod_{i=1}^2 \chi_i \right). \quad (48)$$

For sufficiently high E_{SD}/N_0 , the term $\log(E_{SD}/N_0)$ can be ignored with respect to the dominating term of $(E_{SD}/N_0)^{-2}$. Thus, asymptotically, the second order diversity is achieved, realizing the maximum achievable diversity for the considered scenario with single relay. It should be pointed out that for a non-fading $R \rightarrow D$ link assumption, i.e., h_{RD} , (46) simply reduces to

$$P(\mathbf{X}_{cc} \rightarrow \mathbf{X}_{ee}) \approx 6 \left(\frac{E_{SD}}{N_0} \right)^{-2} \left(\prod_{i=1}^2 \chi_i \right)^{-1}. \quad (49)$$

Comparison of (48) and (49) reveals that, asymptotically, the factor $\log(E_{SD}/N_0)$ incurs a coding gain loss only. It is also interesting to note that (49) has a similar form to the PEP expression derived for conventional STBC with 2 transmit and one receive antenna, c.f., [27, Eq.28].

⁵Due to the underlying assumptions in the derivation, the resulting PEP expression is an approximation.

APPENDIX II

In this Appendix, we present a diversity gain analysis for the mismatched-coherent detector considered in Section V.A. In the following, we consider LMMSE estimator for channel estimation purpose. Assuming perfect power control where $S \rightarrow D$ and $R \rightarrow D$ links are balanced, i.e., $E_{RD}/N_0 = E_{SD}/N_0 \gg 1$, and sufficiently large SNR for the $S \rightarrow R$ link, i.e., $E_{SR}/N_0 \gg E_{SD}/N_0$, we have $\gamma_1 = \gamma_2 \approx 1$. Noting that the channel estimate $\hat{\mathbf{h}}_{LMMSE}$ and the channel estimation error ε_{LMMSE} are uncorrelated [28] and omitting the subscript LMMSE in (37), we can rewrite (32) in scalar form as

$$r_{Dn} = \sum_{i=1}^2 x_{ni} \hat{h}_i + \eta_{Dn}, n = 0, \dots, N-1 \quad (50)$$

where we define $\eta_{Dn} = -\sum_{i=1}^2 x_{ni} \varepsilon_i + n_{Dn}$. In (50), x_{ni} denotes the data symbol transmitted over the i^{th} link at time n . Here, $i = 1, 2$ represent the $S \rightarrow R \rightarrow D$ and $S \rightarrow D$ links, respectively. From the entries of the data matrix $\mathbf{X}$ in (18), it can be seen that $x_{ni} = 0$ for $i = 1$ and $n = 0, 2, \dots, N-2$. Therefore, the variance of the complex Gaussian term η_{Dn} is given as

$$\sigma_{\eta_n}^2 = \begin{cases} E_{SD} \sigma_{\varepsilon_2}^2 + N_0, n = 0, 2, \dots, N-2 \\ E_{SD} \sum_{i=1}^2 \sigma_{\varepsilon_i}^2 + N_0, n = 1, 3, \dots, N-1 \end{cases}$$

Noting that the decoder metric is given by $\sum_{n=1}^N \|r_{Dn} - x_{ni} \hat{h}_i\|^2$, the Chernoff bound on the conditional PEP is then given as

$$P(\mathbf{X}_D \rightarrow \hat{\mathbf{X}}_D | \hat{\mathbf{h}}) \leq \exp \left(-\frac{d^2(\mathbf{X}_D, \hat{\mathbf{X}}_D)}{4\Omega} \right) \quad (51)$$

with

$$\begin{aligned} d^2(\mathbf{X}_D, \hat{\mathbf{X}}_D) &= \sum_{n=1}^N \left\| \sum_{i=1}^2 (x_{ni} - \hat{x}_{ni}) \hat{h}_i \right\|^2 \\ &= \tilde{\lambda}_1 |\hat{h}_1|^2 E_{SD} + \tilde{\lambda}_2 |\hat{h}_2|^2 E_{SD} \end{aligned} \quad (52)$$

where $\tilde{\lambda}_i$, $i = 1, 2$, denote the eigenvalues of $(\mathbf{X}_D - \hat{\mathbf{X}}_D)^H (\mathbf{X}_D - \hat{\mathbf{X}}_D)$ and $\Omega = N_0 + E_{SD} \sum_{i=1}^2 \sigma_{\varepsilon_i}^2 \geq \sigma_{\eta_n}^2$. Notice that $\Omega = \sigma_{\eta_n}^2$, for $n = 1, 3, \dots, N-1$ and $\Omega > \sigma_{\eta_n}^2$ for $n = 0, 2, \dots, N-2$. Defining the covariance matrix $\text{cov}(\hat{\mathbf{h}}) = \text{diag}(|h_{RD}|^2 + \sigma_{\varepsilon_1}^2, 1 + \sigma_{\varepsilon_2}^2)$ for the estimated channel coefficients $\hat{\mathbf{h}} = [\hat{h}_1 \ \hat{h}_2]^T$, we can obtain the unconditional PEP as

$$\begin{aligned} P(\mathbf{X}_D, \hat{\mathbf{X}}_D) &\leq \left(\frac{E_{SD}}{4\Omega} \right)^{-2} \frac{1}{\tilde{\lambda}_1 \tilde{\lambda}_2 (1 + \sigma_{\varepsilon_2}^2)} \\ &\times \exp \left(\frac{4\Omega}{\tilde{\lambda}_1 E_{SD}} + \sigma_{\varepsilon_1}^2 \right) \Gamma \left(0, \frac{4\Omega}{\tilde{\lambda}_1 E_{SD}} + \sigma_{\varepsilon_1}^2 \right) \end{aligned} \quad (53)$$

Under the assumption of $E_{SD}/\Omega \rightarrow \infty$ and using the limiting expression $\lim_{z \rightarrow 0} \Gamma(0, z) \approx -\log(z)$ [20], we can write (53) as

$$P(\mathbf{X}_D, \hat{\mathbf{X}}_D) \leq \left(\frac{E_{SD}}{4\Omega} \right)^{-2} \log \left(\frac{E_{SD}}{N_0} \right) \frac{1}{\tilde{\lambda}_1 \tilde{\lambda}_2 (1 + \sigma_{\varepsilon_2}^2)}. \quad (54)$$

For large E_{SD}/N_0 values, the term $\log(E_{SD}/N_0)$ can be ignored with respect to the dominating term of $(E_{SD}/\Omega)^{-2}$. Thus, asymptotically, the second order diversity is achieved, realizing the maximum achievable diversity for the considered scenario with single relay. Furthermore, we observe that the presence of channel estimation errors in the considered scenario does not affect the diversity order. Similar observations have been earlier reported for conventional space time coding in [28]–[30]. It should be noted that as $P \rightarrow \infty$, we have $\sigma_{\varepsilon_1}^2 = \sigma_{\varepsilon_2}^2 = 0$, thus reducing (53) to

$$P(\mathbf{X}_D, \hat{\mathbf{X}}_D) \leq \frac{1}{\tilde{\lambda}_1 \tilde{\lambda}_2} \left(\frac{E_{SD}}{4N_0} \right)^{-2} \exp \left(\frac{4N_0}{\tilde{\lambda}_1 E_{SD}} \right) \times \Gamma \left(0, \frac{4N_0}{\tilde{\lambda}_1 E_{SD}} \right). \quad (55)$$

Similar result has been reported in [9] for perfect CSI.

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