BER-Optimized Power Allocation for Fading Relay Channels

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***Abstract*—Optimum power allocation is a key technique to realize the full potentials of relay-assisted transmission promised by the recent information-theoretic results. In this paper, we present a comprehensive framework for power allocation problem in a single-relay scenario taking into account the effect of relay location. In particular, we aim to answer the two fundamental questions: Q1) How should the overall transmit power be shared between broadcasting and relaying phases?; Q2) In the relaying phase, how much power should be allocated to relay-to-destination and source-to-destination links? The power allocation problem is formulated to minimize a union bound on the bit error rate (BER) performance assuming amplify-and-forward relaying. We consider both orthogonal and non-orthogonal cooperation protocols. Optimized protocols demonstrate significant performance gains over their original versions which assume equal sharing of overall transmit power between the source and relay terminals as well as between broadcasting and relaying phases. It is observed that optimized virtual (distributed) antenna configurations are able to demonstrate a BER performance as close as 0.4dB within their counterpart co-located antenna configurations.**

***Index Terms*—Distributed space-time codes, pairwise error probability, power allocation, relay channels.**

I. Introduction

THE increasing demand for wireless multimedia and interactive internet services is fuelling intensive research efforts on higher-speed data transmission and improved power efficiency compared to current wireless communication systems. The revolutionary concept of space-time coding [1]-[4] has demonstrated that the deployment of multiple antennas at the transmitter allows for increase in throughput and reliability. Multiple-antenna techniques are very attractive for deployment at base stations in cellular applications and have already been included in the 3rd generation cellular wireless standards and also envisioned for future WLAN standards. Unfortunately, the use of multiple antennas might not be practical at the cellular mobile devices as well as in wireless sensor networks due to size and power constraints. This limitation motivates the concept of cooperative communication between different nodes where a node attempts to use others' antennas to relay its message. User cooperation [5]-[9], also known as cooperative diversity, exploits the broadcast nature of wireless transmission and creates a virtual (distributed) antenna array through cooperating nodes to extract spatial diversity advantages.

The basic ideas behind user cooperation can be traced back to Cover and El Gamal's work on the information theoretic properties of the relay channel [10]. The recent surge of interest in cooperative communication, however, was subsequent to the works of Sendonaris et al. [5], [6] and Laneman et.al. [7], [8]. In [7], Laneman et al. consider a user cooperation scenario where the source signal is transmitted to a destination terminal through $N-1$ half-duplex relay terminals and demonstrate that the receiver achieves a diversity order of N. Their proposed user cooperation protocol is built upon a two-phase transmission scheme. In the first phase (i.e., broadcasting phase), the source broadcasts to the destination and relay terminals. In the second phase (i.e., relaying phase), the relays transmit processed version of their received signals to the destination using either orthogonal subchannels, i.e., repetition based cooperative diversity, or the same subchannel, i.e., space-time coded cooperative diversity. The latter relies on the implementation of conventional orthogonal space-time block coding (STBC) [4] in a distributed fashion among the relay nodes.

The user cooperation protocol considered in [7], [8] effectively realizes receive diversity advantages in a distributed manner and is also known as orthogonal relaying. In [11], Nabar et. al. establish a unified framework of TDMA-based cooperation protocols for single-relay wireless networks. They quantify achievable performance gains for distributed schemes in an analogy to conventional co-located multiantenna configurations. Specifically, they consider three protocols named Protocol I, Protocol II, and Protocol III. In Protocol I, during the first time slot, the source terminal communicates with the relay and destination. During the second time slot, both the relay and source terminals communicate with the destination terminal[1]. Protocol II is the same cooperation protocol proposed by Laneman et.al. in [8]. Protocol III is identical to Protocol I apart from the fact that the destination terminal chooses not to receive the direct source-to-destination transmission during the first time slot for reasons which are possibly imposed from the upper-layer networking protocols (e.g., the destination terminal may be engaged in data transmission to another terminal during the first time slot). It can be noticed from the descriptions of protocols that the signal transmitted to both

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[1]Protocol I with AaF relaying is referred as "non-orthogonal AaF" in [26] and shown to be optimum in terms of diversity-multiplexing trade-off for AaF relaying

the relay and destination terminals is the same over the two time slots in Protocol II. Therefore, classical space-time code construction does not apply to Protocol II. On the other hand, Protocol I and Protocol III can transmit different signals to the relay and destination terminals. Hence, the conventional STBC can be easily applied to these protocols in a distributed fashion[2].

The aforementioned protocols can work either with regenerative (decode-and-forward) or non-regenerative (amplify-and-forward) relaying techniques. In amplify-and-forward (AaF) relaying, the relay terminal re-transmits a scaled version of the received signal without any attempt to decode it. On the other hand, in decode-and-forward (DaF) relaying, the relay terminal decodes its received signal and then re-encodes it (possibly using a different codebook) for transmission to the destination.

Motivation of the paper and related previous work: In pioneering works on user cooperation, the overall transmit power is supposed to be uniformly allocated among the source and relay terminals. Some recent work has shown that the performance of cooperative communication schemes can be substantially improved by optimally distributing the power among cooperating nodes. In [12], Host-Madsen and Zhang derive bounds on ergodic capacity for fading relay channels and study power allocation problem to maximize channel capacity. Their proposed power allocation scheme requires the feedback of channel state information (CSI) of all communication channels to the source for each channel realization. In [13], Ahmed and Aazhang propose a power allocation method relying on partial feedback information. In [14], Qi et.al. investigate power allocation for a two-hop DaF relaying system assuming full CSI availability at the source while the relay has either full or partial CSI. The case of AaF relaying under similar assumptions is studied in Jingmei et.al [15] and extended in [16] for source terminal with multiple antennas. In another paper by Jingmei et.al. [17], power allocation schemes are studied in a multi-cell environment where both DaF or AaF relaying are considered.

Close-loop power allocation schemes require the availability of CSI at the transmitter side and their implementation might be problematic in some practical applications. In [18], Hasna and Alouini investigate the optimal power allocation problem for an open-loop transmission scheme (i.e., CSI information available only at the receiver side) to minimize the outage probability assuming both AaF and DaF relaying. Their results for AaF-relaying are, however, restricted to multi-hop systems without diversity advantages. In [19], [20] Yindi and Hassibi derive an upper bound on the pairwise error probability (PEP) assuming AaF relaying for a large number of relays and minimize PEP bound to formulate optimal power allocation method. They consider a dual-hop scenario in their work. In the broadcasting phase, source sends information to all relays and then stops transmission. In the relaying phase, only the relays forward their received signals to the destination. Under this dual-hop scenario, their conclusion on the optimal power allocation method is that the source uses half the total power and the relays share the other half fairly. For single-relay case, this simply reduces to equal power allocation. It should be emphasized that this conclusion is a result of their implicit underlying assumption that relays are located halfway between source and destination terminals[3]. In [21], Deng et.al. adopt average signal-to-noise ratio (SNR) and outage performance as the optimization metrics and investigate the power allocation problem for Protocol II with AaF relaying. Their proposed method maximizes the sum and product, respectively, of the SNRs in the direct and relaying link and results in improved outage probability performance.

In this paper, we present a comprehensive framework for power allocation problem in open-loop single-relay networks considering Protocols I, II and III of [11] with AaF relaying. Considering bit error rate (BER) as the performance metric and taking into account the effect of relay location, we attempt to answer the following fundamental questions:

- **Q1)** How should overall transmit power be shared between broadcasting and relaying phases?
- **Q2)** How much power should be allocated to relay-to-destination and source-to-destination links[4] in the relaying phase?

For each considered protocol, we propose optimal power allocation methods based on the minimization of a union bound on the BER performance. Optimized protocols demonstrate significant performance gains over their original versions which assume equal sharing of overall transmit power between broadcasting and relay phases and equal sharing of available power in the relaying phase between relay-to-destination and source-to-destination links.

The rest of the paper is organized as follows: In Section II, we introduce the relay-assisted transmission model and describe received signal models for Protocols I, II and III. In Section III, we derive Chernoff bounds on the PEP and calculate union bounds on the BER for each of the protocols. In Section IV, we present the power allocation methods which are optimum in the sense of minimizing BER and discuss their efficiency for various relaying scenarios. In Section V, a comprehensive Monte-Carlo simulation study is presented to demonstrate BER performance of the considered cooperation protocols with equal power allocation and optimum power allocation. The paper finally concludes in Section VI.

Notation: $(.)^*$, $(.)^{\mathrm{T}}$ and $(.)^{\mathrm{H}}$ denote conjugate, transpose, and Hermitian transpose operations, respectively. $E_{y,z}\left(.\right)$ denotes expectation with respect to random variables y and z.

II. TRANSMISSION MODEL

We consider a single-relay scenario where terminals operate in half-duplex mode and are equipped with single transmit and receive antennas. As illustrated in Fig.1, three nodes

[2]It should be noted that the use of STBC has been also proposed by Laneman et.al. in [7] for Protocol II. Their proposed use of STBC implements coding across the relay nodes assuming a scenario with more than one relay and differs from the distributed STBC setup in [11] proposed for Protocol I and Protocol III which involves the source terminal in a single-relay scenario.

[3]In the problem statement of their paper, they mention random relay location, however their transmission model does exclude the effect of relay location.

[4]The presence of source-to-destination transmission in the relaying phase depends on the cooperation protocol. Therefore, Q2 is irrelevant in Protocol II.

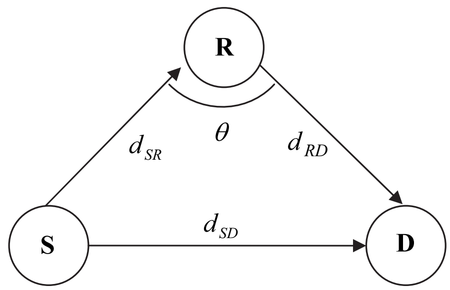

Fig. 1. Relay-assisted transmission model.

source (S), relay (R), and destination (D) are assumed to be located in a two-dimensional plane where d_{SD}, d_{SR}, and d_{RD} denote the distances of source-to-destination (S→D), source-to-relay (S→R), and relay-to-destination (R→D) links, respectively and θ is the angle between lines S→R and R→D. To incorporate the effect of relay geometry in our model, we consider a channel model which takes into account both long-term free-space path loss and short-term Rayleigh fading. The path loss is proportional to $d^{-\alpha}$ where d is the propagation distance and α is path loss coefficient. Typical values of α for various wireless environments can be found in [22]. Assuming the path loss between S→D to be unity, the relative gain of S→R and R→D links are defined as $G_{SR} = (d_{SD}/d_{SR})^{\alpha}$ and $G_{SR} = (d_{SD}/d_{RD})^{\alpha}$ [23]. They can be further related to each other by $G_{SR}^{-2/\alpha} + G_{RD}^{-2/\alpha} - 2G_{SR}^{-1/\alpha}G_{RD}^{-1/\alpha}\cos\theta = 1$ through law of cosines.

A. Protocol I

In Protocol I, the source terminal communicates with the relay and destination during the first time slot. In the second time slot, both the relay and source terminals communicate with the destination terminal. Let x_1 denote the transmitted signal in the first time slot. We assume x_1 is the output of a MPSK modulator with unit energy. Considering path-loss effects, the received signals at the relay and destination are given as

$$r_R = \sqrt{2G_{SR}K_T E}\,h_{SR}x_1 + n_R, \tag{1}$$

$$r_{D1} = \sqrt{2K_T E}\,h_{SD}x_1 + n_{D1}, \tag{2}$$

where n_R and n_{D1} are the independent samples of zero-mean complex Gaussian random variables with variance $N_0/2$ per dimension, which model the additive noise terms. h_{SR} and h_{SD} denote the zero-mean complex Gaussian fading coefficients with variances 0.5 per dimension, leading to a Rayleigh fading channel assumption. Here, the total energy (to be used by both source and relay terminals) is $2E$ during two time slots yielding an average power in proportion to E per time slot, i.e., assuming unit time duration. K_T is an optimization parameter and controls the fraction of power which is reserved for the source terminal's use in the first time slot, i.e., broadcasting phase. The relay terminal normalizes the received signal r_R by a factor of

$$\sqrt{E_{n_R,h_{SR}}[|r_R|^2]} = \sqrt{2G_{SR}K_T E + N_0}, \tag{3}$$

where we have used $E_{h_{SR}}[|h_{SR}|^2] = 1$ and $E_{n_R}[|n_R|^2] = N_0$. The relay re-transmits the signal during the second time slot. The source terminal simultaneously transmits x_2 using $2(1-K_T)K_S E$ where K_S is another optimization parameter and controls the fraction of power which is reserved for the source terminal's use in the second time slot, i.e., relaying phase. Therefore, the power used by the source in broadcasting and relaying phase is, respectively, $2K_T E$ and $2(1-K_T)K_S E$. Power used by the relay terminal is $2(1-K_T)(1-K_S)E$.

The received signal at the destination terminal is the superposition of transmitted signals by the relay and source terminals resulting in

$$\begin{aligned} r_{D2} = {} & \sqrt{\frac{4G_{SR}G_{RD}K_T(1-K_T)(1-K_S)E^2}{2G_{SR}K_T E + N_0}}\,h_{RD}h_{SR}x_1 \\ & + \sqrt{2(1-K_T)K_S E}\,h_{SD}x_2 + \tilde{n}_{D2}, \end{aligned} \tag{4}$$

where we define the effective noise term as

$$\tilde{n}_{D2} = \sqrt{\frac{2G_{RD}(1-K_T)(1-K_S)E}{[2G_{SR}K_T E + N_0]}}\,h_{RD}n_R + n_{D2}. \tag{5}$$

In the above, n_{D2} is modelled as a zero-mean complex Gaussian random variable with variance $N_0/2$ per dimension. h_{RD} is a zero-mean complex Gaussian fading coefficient with variance 0.5 per dimension, leading to a Rayleigh fading channel assumption similar to h_{SR} and h_{SD}. Conditioned on h_{RD}, $\tilde{n}_{D2}$ turns out to be complex Gaussian. We assume that the destination terminal normalizes the received signal given by (4) with $\sqrt{1 + \frac{2G_{RD}(1-K_T)(1-K_S)E|h_{RD}|^2}{(2G_{SR}K_T E + N_0)}}$ [5], resulting in

$$\bar{r}_{D2} = \sqrt{A_1}\sqrt{E}\,h_{RD}h_{SR}x_1 + \sqrt{A_2}\sqrt{E}\,h_{SD}x_2 + \bar{n}_{D2}, \tag{6}$$

where $\bar{n}_{D2}$ is a complex Gaussian random variable which has zero mean and variance of $N_0/2$ per dimension. In (6), A_1 and A_2 are defined as $A_1 = A_{N1}\Big/\left(A_D + |h_{RD}|^2\right)$ and $A_2 = A_{N2}\Big/\left(A_D + |h_{RD}|^2\right)$, respectively, where $A_{N1} = 2G_{SR}K_T$, $A_{N2} = K_S(1 + 2G_{SR}K_T SNR)/[G_{RD}(1-K_S)SNR]$, and $A_D = [1 + 2G_{SR}K_T SNR]/[2G_{RD}(1-K_T)(1-K_S)SNR]$ with $SNR = E/N_0$.

After setting up the relay-assisted transmission model for Protocol I given by (2) and (6), we now introduce space-time coding across the transmitted signals x_1 and x_2. For the case of single relay deployment as considered here, we use STBC designed for two transmit antennas, i.e., Alamouti's scheme [3]. The received signals at the destination terminal during the four time slots can be written in a compact matrix form as $\mathbf{r} = \mathbf{hX} + \mathbf{n}$ where $\mathbf{h} = \begin{bmatrix} h_{SD} & h_{SR}h_{RD} \end{bmatrix}$, $\mathbf{n} = \begin{bmatrix} n_{D1} & \bar{n}_{D2} & n_{D3} & \bar{n}_{D4} \end{bmatrix}$ and

$$\mathbf{X} = \begin{bmatrix} \sqrt{A_0 E}x_1 & \sqrt{A_2 E}x_2 & \sqrt{A_0 E}x_2 & \sqrt{A_2 E}x_1^* \\ 0 & \sqrt{A_1 E}x_1 & 0 & -\sqrt{A_1 E}x_2^* \end{bmatrix}. \tag{7}$$

Each entry of $\mathbf{n}$ is a zero-mean complex Gaussian random variable and $A_0 = 2K_T$. Since distributed implementation of repetition code offers the same rate of Alamouti code in

[5]This does not change the signal-to-noise ratio, but simplifies the ensuing presentation [11]

the considered single-relay scenario[6], we also consider it as a possible candidate for the underlying distributed code. For the repetition code, $\mathbf{X}$ is given by

$$\mathbf{X} = \begin{bmatrix} \sqrt{A_0 E}x_1 & \sqrt{A_2 E}x_1 & \sqrt{A_0 E}x_2 & \sqrt{A_2 E}x_2 \\ 0 & \sqrt{A_1 E}x_1 & 0 & \sqrt{A_1 E}x_2 \end{bmatrix}. \tag{8}$$

B. Protocol II

Protocol II realizes receive diversity in a distributed manner and does not involve transmit diversity. Therefore, unlike Protocol I which relies on two optimization parameters K_T and K_S, only K_T is relevant for Protocol II optimization. Let x_1 denote the transmitted signal. Considering path-loss effects, the received signals at the relay and destination are given as

$$r_R = \sqrt{2G_{SR}K_T E}h_{SR}x_1 + n_R, \tag{9}$$

$$r_{D1} = \sqrt{2K_T E}h_{SD}x_1 + n_{D1}. \tag{10}$$

There is no source-to-destination transmission in the second time slot. The received signal at destination is given by

$$r_{D2} = \sqrt{\frac{[4G_{SR}G_{RD}K_T(1-K_T)E^2]}{[2G_{SR}K_T E + N_0]}}h_{RD}h_{SR}x_1 + \tilde{n}_{D2}, \tag{11}$$

where the effective noise term is defined as

$$\tilde{n}_{D2} = \sqrt{\frac{[2G_{RD}(1-K_T)E]}{[2G_{SR}K_T E + N_0]}}h_{RD}n_R + n_{D2}. \tag{12}$$

$\tilde{n}_{D2}$ is complex Gaussian conditioned on h_{RD}. In a similar manner to the previous section, we normalize (11) such that additive noise term has a variance of N_0 which yields

$$\bar{r}_{D2} = \sqrt{B_1}\sqrt{E}h_{SR}h_{RD}x_1 + \bar{n}_{D2}, \tag{13}$$

where we define $B_1 = B_N\Big/\left(B_D + |h_{RD}|^2\right)$ with $B_D = [1 + 2G_{SR}K_T SNR]/[2G_{RD}(1-K_T)SNR]$ and $B_N = 2G_{SR}K_T$. (10) and (13) can be written in matrix form as in the previous section where $\mathbf{X}$ now has the form of

$$\mathbf{X} = \begin{bmatrix} \sqrt{B_0 E}x_1 & 0 \\ 0 & \sqrt{B_1 E}x_1 \end{bmatrix}, \tag{14}$$

with $B_0 = 2K_T$.

C. Protocol III

Protocol III is identical to Protocol I apart from the fact that the destination terminal chooses not to receive the direct source-to-destination transmission during the first time slot for reasons which are possibly imposed from the upper-layer networking protocols. For example, the destination terminal may be engaged in data transmission to another terminal during the first time slot. Following similar steps as in Section II.A for Protocol I, the received signals can be written in matrix form where $\mathbf{X}$ is now given by

$$\mathbf{X} = \begin{bmatrix} \sqrt{A_2 E}x_2 & \sqrt{A_2 E}x_1^* \\ \sqrt{A_1 E}x_1 & -\sqrt{A_1 E}x_2^* \end{bmatrix}. \tag{15}$$

[6]In distributed implementation of single-relay transmission, Alamouti's code is able to transmit two symbols in four time intervals resulting in a rate of 1/2 [11]

For the repetition code, $\mathbf{X}$ takes the form of

$$\mathbf{X} = \begin{bmatrix} \sqrt{A_2 E}x_1 & \sqrt{A_2 E}x_2 \\ \sqrt{A_1 E}x_1 & -\sqrt{A_1 E}x_2 \end{bmatrix}. \tag{16}$$

III. Union Bound on the BER performance

We consider BER performance as our objective function for power allocation problem under consideration. A union bound on the BER for coded systems is given by [24, Ch.12, p. 653]

$$P_b \le \frac{1}{n}\sum_{\mathbf{X}} p(\mathbf{X}) \sum_{\mathbf{X}\ne\hat{\mathbf{X}}} q\left(\mathbf{X}\to\hat{\mathbf{X}}\right) P\left(\mathbf{X}\to\hat{\mathbf{X}}\right), \tag{17}$$

where $p(\mathbf{X})$ is the probability that codeword $\mathbf{X}$ is transmitted, $q\left(\mathbf{X}\to\hat{\mathbf{X}}\right)$ is the number of information bit errors in choosing another codeword $\hat{\mathbf{X}}$ instead of the original one, and n is the number of information bits per transmission. In (17), $P\left(\mathbf{X}\to\hat{\mathbf{X}}\right)$ is the probability of deciding in favour of $\hat{\mathbf{X}}$ instead of $\mathbf{X}$ and called as pairwise error probability (PEP). As reflected by (17), PEP is the building block for the derivation of union bounds to the error probability.

In this section, we derive PEP expressions for each protocol under consideration. A Chernoff bound on the (conditional) PEP is given by [2]

$$P\left(\mathbf{X}\to\hat{\mathbf{X}}\,|\mathbf{h}\right) \le \exp\left(\frac{-d^2\left(\mathbf{X},\hat{\mathbf{X}}\,|\mathbf{h}\right)}{4N_0}\right), \tag{18}$$

where the Euclidean distance (conditioned on fading channel coefficients) between $\mathbf{X}$ and $\hat{\mathbf{X}}$ is $d^2\left(\mathbf{X},\hat{\mathbf{X}}\,|\mathbf{h}\right) = \mathbf{h}\boldsymbol{\Delta}\mathbf{h}^{\mathrm{H}}$ with $\boldsymbol{\Delta} = (\mathbf{X}-\hat{\mathbf{X}})(\mathbf{X}-\hat{\mathbf{X}})^{\mathrm{H}}$. Recalling the definitions of $\mathbf{X}$ in (7), (14), (15) for different protocols and carrying out the expectation with respect to $\mathbf{h}$, we obtain PEP expressions for Protocols I, II and III in the following:

A. PEP for Protocol I

Replacing (7) in (18), we have

$$P\left(\mathbf{X}\to\hat{\mathbf{X}}\,|\mathbf{h}\right) \le e^{-\left\{\frac{SNR\chi_1}{4}\left[(A_0+A_2)|h_{SD}|^2 + A_1|h_{SR}|^2|h_{RD}|^2\right]\right\}}, \tag{19}$$

with $\chi_1 = |x_1 - \hat{x}_1|^2 + |x_2 - \hat{x}_2|^2$. Averaging (19) with respect to $|h_{SR}|^2$ and $|h_{SD}|^2$ which follow exponential distribution, we obtain

$$P\left(\mathbf{X}\to\hat{\mathbf{X}}\,|h_{RD}\right) \le \left(1 + \frac{SNR(A_0+A_2)\chi_1}{4}\right)^{-1} \left(1 + \frac{SNRA_1\chi_1}{4}|h_{RD}|^2\right)^{-1}. \tag{20}$$

After some mathematical manipulation, we obtain

$$P\left(\mathbf{X}\to\hat{\mathbf{X}}\middle| h_{RD}\right) \le \delta_1\left(1 + \alpha_1\frac{1}{|h_{RD}|^2 + \lambda_1} + \beta_1\frac{1}{|h_{RD}|^2 + \mu_1}\right). \tag{21}$$

Here, δ_1, λ_1, μ_1, α_1 and β_1 are defined, respectively, as

$$\delta_1 = \left(1 + \frac{SNR}{4}A_{N1}\chi_1\right)^{-1}\left(1 + \frac{SNR}{4}A_0\chi_1\right)^{-1}, \tag{22}$$

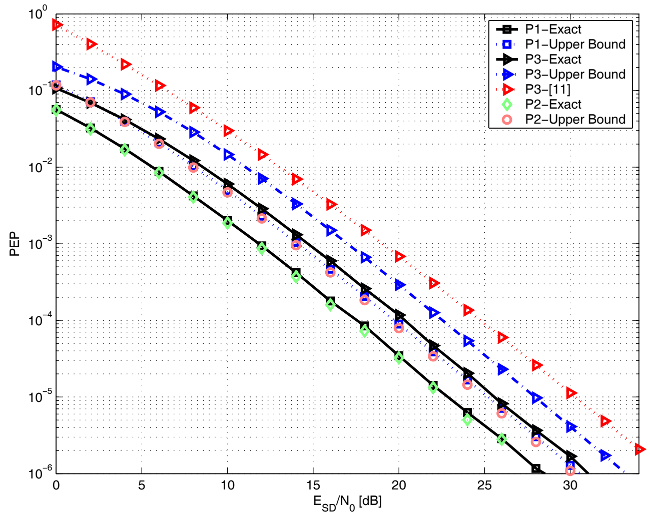

Fig. 2. Comparison of exact and derived upper bounds on PEP.

$$\lambda_1 = \frac{A_D\left(1+\frac{SNR}{4}A_0\chi_1\right)+\frac{SNR}{4}A_{N2}\chi_1}{1+\frac{SNR}{4}A_0\chi_1}, \tag{23}$$

$$\mu_1 = A_D \Big/ \left(1+\frac{SNR}{4}A_{N1}\chi_1\right), \tag{24}$$

$$\alpha_1 = \frac{-\left(2A_D-\lambda_1-\mu_1\right)\lambda_1 + A_D^2 - \lambda_1\mu_1}{\mu_1-\lambda_1}, \tag{25}$$

$$\beta_1 = \frac{-\left(2A_D-\lambda_1-\mu_1\right)\mu_1 + A_D^2 - \lambda_1\mu_1}{\lambda_1-\mu_1}, \tag{26}$$

where A_0, A_D, A_{N1}, and A_{N2} have been earlier introduced in our transmission model. Carrying out an expectation of (21) with respect to $|h_{RD}|^2$, we have (27) which is given at the top of the next page. Eq. (27) has a similar form of [25- p.366, 3.384] and readily yields a closed-form solution as,

$$P\left(\mathbf{X}\to\hat{\mathbf{X}}\right) \le \Psi_1(\chi_1) \hat{=}\delta_1\left[1+\alpha_1\exp(\lambda_1)\Gamma(0,\lambda_1)+\beta_1\exp(\mu_1)\Gamma(0,\mu_1)\right], \tag{28}$$

where $\Gamma(.,.)$ denotes the incomplete gamma function [25]. It has been verified through a Monte-Carlo simulation that for various values of SNR and relay location the derived upper bound given by (28) lies within $\sim$ 2.3dB of the exact PEP expression (see Fig.2). Assuming equal-power allocation case, (i.e., $K_T = 0.5$ and $K_S = 0.5$), equal distances among all nodes (i.e., $G_{SR} = G_{SD} = G_{RD} = 1$), and sufficiently high SNR, (28) reduces to

$$P\left(\mathbf{X}\to\hat{\mathbf{X}}\right) \le \left(\frac{SNR\chi_1}{4}\right)^{-2} \times\left[1.2+\frac{4}{3}\exp\left(\frac{8}{SNR\chi_1}\right)\Gamma\left(0,\frac{8}{SNR\chi_1}\right)\right] \tag{29}$$

which illustrates that a diversity order of two is achievable.

B. PEP for Protocol II

Replacing (14) in (18) and averaging the resulting expression with respect to $|h_{SR}|^2$ and $|h_{SD}|^2$, we have

$$P\left(\mathbf{X}\to\hat{\mathbf{X}}\,|h_{RD}\right) \le \left(1+\frac{SNRB_0\chi_2}{4}\right)^{-1} \times\left(1+\frac{SNRB_1\chi_2}{4}|h_{RD}|^2\right)^{-1}, \tag{30}$$

with $\chi_2 = |x_1-\hat{x}_1|^2$. After some mathematical manipulation, we obtain

$$P\left(\mathbf{X}\to\hat{\mathbf{X}}\,|h_{RD}\right) \le \delta_2\left(1+\frac{\beta_2}{|h_{RD}|^2+\lambda_2}\right). \tag{31}$$

Here, δ_2, λ_2 and $beta_2$ are defined as

$$\delta_2 = \left(1+\frac{SNR}{4}B_0\chi_2\right)^{-1}\left(1+\frac{SNR}{4}B_N\chi_2\right)^{-1}, \tag{32}$$

$$\lambda_2 = B_D\Big/\left(1+\frac{SNR}{4}B_N\chi_2\right). \tag{33}$$

$$\beta_2 = B_D - \lambda_2, \tag{34}$$

where B_0, B_D, and B_N have been earlier introduced in our transmission model. By averaging (31) over $|h_{RD}|^2$, we obtain the final form for PEP as

$$P\left(\mathbf{X}\to\hat{\mathbf{X}}\right) \le \Psi_2(\chi_2)\hat{=}\delta_2\left[1+\beta_2\exp(\lambda_2)\Gamma(0,\lambda_2)\right]. \tag{35}$$

Similar to the upper bound derived for Protocol I, this upper bound also lies within $\sim$ 2.3dB of the exact PEP expression. Under the assumptions of equal-power allocation, equal distances among all nodes, and sufficiently high SNR, (35) simplifies to

$$P\left(\mathbf{X}\to\hat{\mathbf{X}}\right) \le \left(\frac{SNR\chi_2}{4}\right)^{-2} \times\left[1+\exp\left(\frac{4}{SNR\chi_2}\right)\Gamma\left(0,\frac{4}{SNR\chi_2}\right)\right], \tag{36}$$

which illustrates that a diversity order of two is extracted. It should be further noted that if we use non-fading h_{RD} assumption (i.e., $h_{RD} = 1$) as in [21], the final PEP has a similar form of (30). In this case, to minimize the resulting PEP, we need to maximize the summation of the sum and product of the SNRs in the direct and relaying links. This is related to the criteria in [21] which aim to maximize either the sum or the product of the SNRs.

C. PEP for Protocol III

Replacing (15) in (18) and averaging the resulting expression with respect to $|h_{SR}|^2$ and $|h_{SD}|^2$, we have

$$P\left(\mathbf{X}\to\hat{\mathbf{X}}\,|h_{RD}\right) \le \left(1+\frac{SNRA_2\chi_3}{4}\right)^{-1} \times\left(1+\frac{SNRA_1\chi_3}{4}|h_{RD}|^2\right)^{-1}. \tag{37}$$

$$P\left(\mathbf{X} \to \hat{\mathbf{X}}\right) \leq \delta_1 \left[1 + \alpha_1 \int_0^\infty \frac{1}{|h_{RD}|^2 + \lambda_1} \exp\left(-|h_{RD}|^2\right) d|h_{RD}|^2 + \beta_1 \int_0^\infty \frac{1}{|h_{RD}|^2 + \mu_1} \exp\left(-|h_{RD}|^2\right) d|h_{RD}|^2\right]. \tag{27}$$

After some mathematical manipulation, we obtain

$$P\left(\mathbf{X} \to \hat{\mathbf{X}}\middle| h_{RD}\right) \leq \delta_3 \left(1 + \alpha_3 \frac{1}{|h_{RD}|^2 + \lambda_3} + \beta_3 \frac{1}{|h_{RD}|^2 + \mu_3}\right) \tag{38}$$

where δ_3, λ_3, μ_3, α_3, and β_3 are defined as

$$\delta_3 = \left(1 + \frac{SNR}{4} A_{N1} \chi_3\right)^{-1}, \tag{39}$$

$$\lambda_3 = A_D + \frac{SNR}{4} A_{N2} \chi_3, \tag{40}$$

$$\mu_3 = A_D \Big/ \left(1 + \frac{SNR}{4} A_{N1} \chi_3\right), \tag{41}$$

$$\alpha_3 = \frac{-\left(2A_D - \lambda_3 - \mu_3\right)\lambda_3 + A_D^2 - \lambda_3 \mu_3}{\mu_3 - \lambda_3}, \tag{42}$$

$$\beta_3 = \frac{-\left(2A_D - \lambda_3 - \mu_3\right)\mu_3 + A_D^2 - \lambda_3 \mu_3}{\lambda_3 - \mu_3}. \tag{43}$$

By averaging (38) over $|h_{RD}|^2$, we obtain the final form for PEP as

$$P\left(\mathbf{X}, \hat{\mathbf{X}}\right) \leq \Psi_3\left(\chi_3\right) \hat{=} \delta_3 \left[1 + \alpha_3 \exp\left(\lambda_3\right) \Gamma\left(0, \lambda_3\right) + \beta_3 \exp\left(\mu_3\right) \Gamma\left(0, \mu_3\right)\right]. \tag{44}$$

with $\chi_3 = |x_1 - \hat{x}_1|^2 + |x_2 - \hat{x}_2|^2$ [7]. The tightness of upper bound given by (44) is similar to that of Protocol I (See Fig.2). For comparison purpose, we also include the plot of PEP expression derived in [11]. Our derived PEP is 2dB tighter than the one of [11]. Under the assumption of equal-power allocation, high SNR, and equal distances among all nodes, (44) simplifies to

$$P\left(\mathbf{X} \to \hat{\mathbf{X}}\right) \leq \left(\frac{SNR\chi_3}{8}\right)^{-2} \exp\left(\frac{8}{SNR\chi_3}\right) \times \Gamma\left(0, \frac{8}{SNR\chi_3}\right), \tag{45}$$

which shows that a diversity order of two is available. For $G_{SR}/G_{RD} << 1$, i.e., relay is close to destination, it can be shown that (45) reduces to

$$P\left(\mathbf{X} \to \hat{\mathbf{X}}\right) \leq \left(\frac{SNR\chi_3}{4}\right)^{-1}. \tag{46}$$

This demonstrates that Protocol III with equal power allocation suffers diversity loss for a scenario where the relay is close to destination. We will later demonstrate that optimum power allocation guarantees full diversity for Protocol III regardless of the relay location.

[7] Both Protocols I and III are built upon Alamouti code. Therefore, $\chi_1 = \chi_3$.

IV. Optimum Power Allocation

As noted in Section III, the objective function in our optimization problem is the union bound on BER. Replacing PEP expressions given by (28), (35), (44), respectively, for Protocols I, II and III, in the BER bound given by (17), we obtain the objective functions to be used for power allocation. The specific form of BER expressions depends on the modulation scheme and underlying code. For example, if BPSK is used as the modulation scheme, upper bounds on BER scheme can be calculated as

$$P_{b1} \leq \Psi_1\left(\chi_1 = 2\right) + \Psi_1\left(\chi_1 = 4\right), \tag{47}$$

$$P_{b2} \leq \Psi_2\left(\chi_2 = 4\right), \tag{48}$$

$$P_{b3} \leq \Psi_3\left(\chi_3 = 2\right) + \Psi_3\left(\chi_3 = 4\right), \tag{49}$$

for Protocols I, II, and III respectively. If QPSK is used, the upper bounds on BER are given as

$$P_{b1} \leq \Psi_1\left(\chi_1 = 2\right) + 3\Psi_1\left(\chi_1 = 4\right) + 3\Psi_1\left(\chi_1 = 6\right) + \Psi_1\left(\chi_1 = 8\right), \tag{50}$$

$$P_{b2} \leq \Psi_2\left(\chi_2 = 2\right) + \Psi_2\left(\chi_2 = 4\right), \tag{51}$$

$$P_{b3} \leq \Psi_3\left(\chi_3 = 2\right) + 3\Psi_3\left(\chi_3 = 4\right) + 3\Psi_3\left(\chi_3 = 6\right) + \Psi_3\left(\chi_3 = 8\right). \tag{52}$$

Similar bounds can be easily found for higher order PSK schemes. We need to minimize the resulting BER expressions with respect to the power allocation parameters K_T and K_S ($0 \leq K_T, K_S \leq 1$). These expressions are found to be convex functions with respect to optimization parameters K_T and K_S. Convexity of the functions under consideration guarantees that local minimum found through optimization will be indeed a global minimum. Unfortunately, an analytical solution for power allocation values in the general case is very difficult, if not infeasible. In the rest, we follow two approaches: First, we pursue numerical optimization of union BER bounds to find out the optimal values of K_T and K_S. For this purpose, we have used Matlab optimization toolbox command “fmincon” designed to find the minimum of a given constrained nonlinear multivariable function. Second, we impose certain assumptions on the relay locations, consider non-fading R→D link, and derive optimal allocation values analytically for Protocols II and III based on the simplified PEPs. Our results demonstrate that analytical solutions largely coincide with numerical results although the former have been obtained under some simplifying assumptions.

Under non-fading R→D channel assumption (i.e., $h_{RD} = 1$), optimum value of K_T for Protocol II can be found by differentiating (30) and equating it to zero. Assuming $G_{SR} \approx 1$ and $G_{RD} >> 1$ (i.e., relay is close to destination), we find

$$K_T = -\frac{\sqrt{\left(2G_{RD}SNR+1\right)\left(8G_{RD}^2+16G_{RD}^2 SNR+2G_{RD}SNR+1\right)}}{8G_{RD}SNR(G_{RD}-1)} + \frac{(4G_{RD}-1)(2G_{RD}SNR+1)}{8G_{RD}SNR(G_{RD}-1)}. \tag{53}$$

Under the assumption of $G_{SR}/G_{RD} = 0$dB (i.e., relay is equidistant from source and destination), we have

$$K_T = \frac{2SNR - 1 + \sqrt{1.18 + 3.5SNR + 4SNR^2}}{6SNR}. \quad (54)$$

Under non-fading R→D channel assumption, optimum values of K_T and K_S for Protocol III can be found by differentiating (37) and equating it to zero. Assuming large values of SNR, $G_{SR} \approx 1$ and $G_{RD} >> 1$, the optimum values are

$$K_T = \frac{3G_{RD} - \sqrt{G_{RD}(G_{RD}+8)}}{4(G_{RD}-1)}, \quad (55)$$

$$K_S = \frac{4 - G_{RD} + \sqrt{G_{RD}(G_{RD}+8)}}{8}. \quad (56)$$

Under the assumption of $G_{SR} >> 1$ and $G_{RD} \approx 1$(i.e., relay is close to source), we have

$$K_T = \frac{2}{3 + \sqrt{8G_{SR}+1}}, \quad (57)$$

$$K_S = \frac{4G_{SR} - 1 + \sqrt{8G_{SR}+1}}{8G_{SR}}. \quad (58)$$

For the particular case of $G_{SR}/G_{RD} = 0$dB (i.e., the relay is equidistant from source and destination terminals), we obtain $K_T = 1/3$ and $K_S = 3/4$. Finally, we note that an analytical solution for Protocol I is intractable even under the considered simplifying assumptions.

In Table I, we present optimum values of K_T and K_S (obtained through numerical optimization) for various values of G_{SR}/G_{RD} which reflects the effect of relay location. More negative this ratio is, more closely the relay is placed to destination terminal. On the other hand, positive values of this ratio indicate that the relay is more close to source terminal. For Protocol I, we observe from Table I(a) that

- When the relay is close to destination, optimum values of K_T are ~ 0.95, and those of K_S are ~ 0. These values indicate that it is better to spend most of power in broadcast phase, and in the relaying phase available power (i.e., $1 - K_T$) should be dedicated to the relay terminal.
- When relay is equidistant from source and destination, the optimum value of K_T is $\sim 2/3$ which means that 66% of power should be spent in the broadcast phase. The optimum value of K_S is still ~ 0 which indicates that all available power should be dedicated to the relay terminal in the relaying phase.
- When relay is close to source and system is operating in low SNR region ($0 - 10$dB), optimum values of K_T and K_S are the same as in previous case, but in higher SNR region (> 10dB) the optimum value of K_S increases with increasing SNR while that of K_T decreases.

For Protocol II, we observe from Table I(b) that

- When relay is equidistant or close to source, $\sim 66\%$ of power is required by the source to achieve optimum performance. This perfectly matches to the analytical result obtained from (54).
- When relay is close to destination, $\sim 95\%$ of power should be used in broadcast phase. This can be readily compared to (53) which yields very similar results. For example, for $G_{SR}/G_{RD} = -30$dB and $SNR = 20$dB, K_T is equal to 0.97.

For Protocol III, we observe from Table I(c) that

- Optimum values of K_S and K_T are ~ 1 and ~ 0.5 for negative values of G_{SR}/G_{RD} ratio (in dB). This is in contrast with small values of observed for Protocol I. Here, it should be noted that Protocol I is able to guarantee a diversity order of two even with equal power allocation owing to the existence of S→D link in the relaying phase. However, the diversity order of Protocol III with equal power allocation reduces to one for scenarios where relay is close to destination. Such large values of K_S in the optimized Protocol III aim to balance the S→D and R→D links so that diversity order of two can be extracted, guaranteeing the full diversity. We also note that our analytical derivations give similar results to those obtained through numerical optimization. For example, for $G_{SR}/G_{RD} = -30$dB, (55) and (56) yield $K_S = 0.99$ and $K_T = 0.49$.
- For equidistant nodes, optimum values of K_T and K_S through numerical optimization are found to be ~ 0.26 and ~ 0.75, respectively. These results are also in line with our analytical derivations for this particular relay location.
- When relay is close to source, numerical optimization yields $K_T \sim 0$ and $K_S \sim 0.6$. These are similar to our analytical results which can be obtained from (57) and (58). For example, assuming $G_{SR}/G_{RD} = 30$dB, (57) and (58) yield $K_T = 0.2$ and $K_S = 0.51$.

TABLE I
POWER ALLOCATION PARAMETERS FOR DISTRIBUTED ALAMOUTI CODE.

(a) Protocol I

SNR	G_{SR}/G_{RD} -30dB		0dB		30dB	
[dB]	K_T	K_S	K_T	K_S	K_T	K_S
5	0.9535	0.0000	0.6648	0.0000	0.6336	0.0000
10	0.9516	0.0000	0.6501	0.0000	0.6153	0.0000
15	0.9503	0.0000	0.6417	0.0000	0.5812	0.0586
20	0.9493	0.0000	0.6358	0.0000	0.3680	0.3652
25	0.9486	0.0000	0.6315	0.0000	0.3682	0.3583
30	0.9479	0.0000	0.6280	0.0000	0.3608	0.3599

(b) Protocol II

SNR	G_{SR}/G_{RD} -30dB	0dB	30dB
[dB]	K_T	K_T	K_T
5	0.9551	0.6728	0.6466
10	0.9530	0.6580	0.6267
15	0.9517	0.6493	0.6156
20	0.9507	0.6432	0.6081
25	0.9499	0.6385	0.6025
30	0.9492	0.6348	0.5982

(c) Protocol III

SNR	G_{SR}/G_{RD} -30dB		0dB		30dB	
[dB]	K_T	K_S	K_T	K_S	K_T	K_S
5	0.9276	0.0722	0.2697	0.7707	0.0236	0.6532
10	0.4765	0.9984	0.2660	0.7565	0.0223	0.6034
15	0.4780	0.9984	0.2631	0.7484	0.0433	0.6066
20	0.4787	0.9984	0.2609	0.7427	0.0464	0.5999
25	0.4792	0.9984	0.2590	0.7384	0.0490	0.5950
30	0.4795	0.9983	0.2575	0.7349	0.0515	0.5911

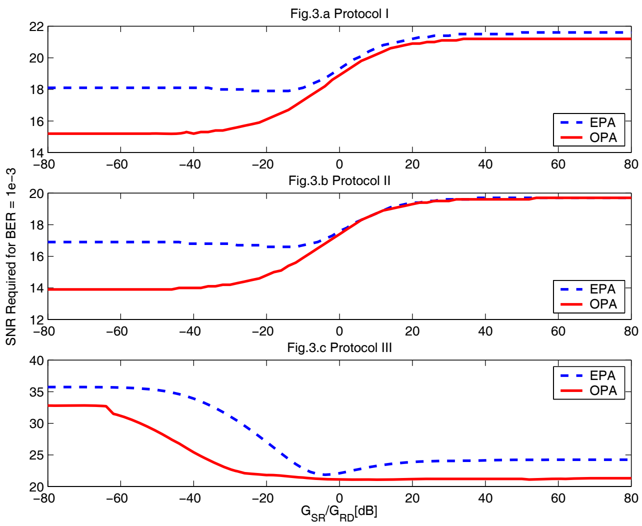

Fig. 3. SNR required to achieve BER of 10^{-3} for Protocols I, II and III.

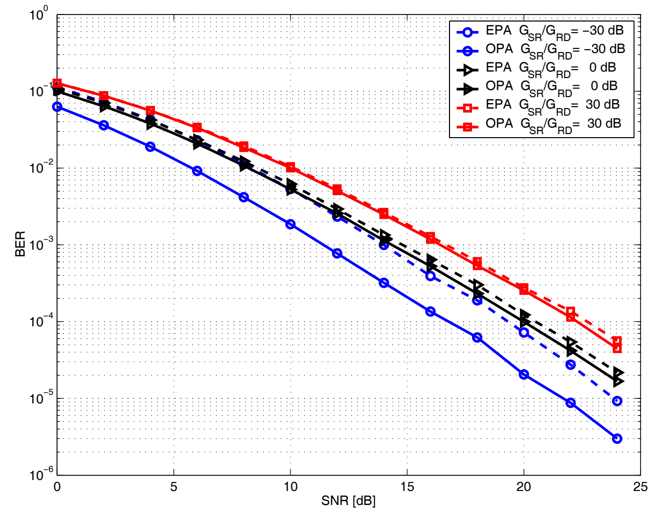

Fig. 4. Simulated BER performance of Protocol I for different values of G_{SR}/G_{RD}.

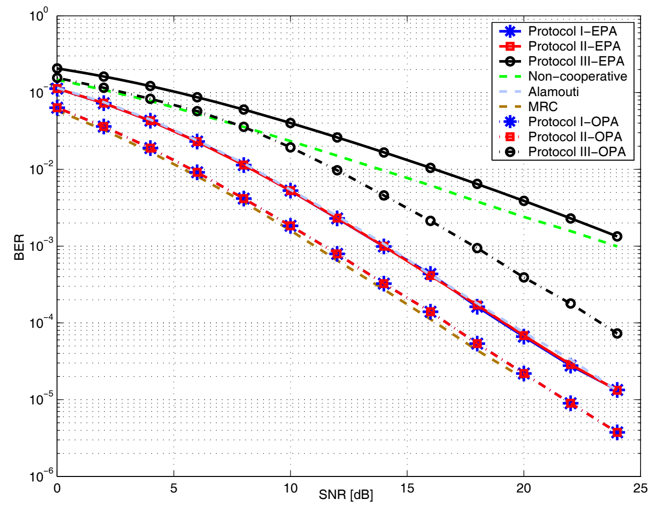

Fig. 5. Performance comparison of Protocols I, II and III with EPA and OPA ($G_{SR}/G_{RD} = -30$dB).

In Fig. 3, we demonstrate performance gains in power efficiency (as predicted by the derived PEP expressions) achieved by optimum power allocation (OPA) over equal power allocation (EPA) for a target BER of 10^{-3} assuming QPSK modulation. The performance gains are presented as a function of G_{SR}/G_{RD}. In Fig.3.a given for Protocol I, we observe performance improvements of $\sim$ 0.4dB and 0.3dB at $G_{SR}/G_{RD} = 0$dB and $G_{SR}/G_{RD} = 30$dB, respectively. Advantages of OPA are more pronounced for negative values of G_{SR}/G_{RD}. For example, an improvement of $\sim$ 2.5dB is observed for $G_{SR}/G_{RD} = -30$dB. It is clear from this figure that although power optimization helps in all cases, it is more rewarding in scenarios where relay is close to destination. In Fig.3.b given for Protocol II, we observe performance improvements up to $\sim$ 2.6dB for negative values of G_{SR}/G_{RD}. For positive values, it is observed that OPA and EPA performance curves converge to each other. In Fig.3.c given for Protocol III, we observe significant performance improvements for both negative and positive G_{SR}/G_{RD} values. In particular, the performance improvements are $\sim$ 8.4dB and $\sim$ 2.9dB at $G_{SR}/G_{RD} = -30$dB and $G_{SR}/G_{RD} = 30$dB, respectively. The change in characteristic behaviour of Protocol III in comparison to those of Protocols I and II should be also noted. This is actually not unexpected; recall that Protocol III realizes a distributed transmit diversity scheme, so it is expected to perform good when relay is close to source mimicking a virtual transmit antenna array. Protocol II implements receive diversity, so it is expected to perform good when relay is close to destination mimicking a virtual receive antenna array. Protocol I is a combination of both Protocol II and Protocol III. It is observed from our results that the advantages of receive diversity are dominating in this hybrid version.

V. Simulation Results

To further confirm the performance gains of OPA promised by the derived expressions, we have conducted a Monte Carlo simulation study to compare the BER performance of the considered protocols with EPA and OPA. Our simulation results for Protocol I are presented in Fig. 4 where we assume QPSK modulation and $\theta = \pi$. We observe performance improvements of 2.5dB, 0.4dB, and 0.29dB at a target BER of 10^{-3} for $G_{SR}/G_{RD} = -30$dB, 0dB and 30dB respectively. These are similar to performance gains predicted for Protocol I through our PEP expressions. Similar confirmation holds for the other two protocols and those simulation results are not included here due to the space limitations.

Fig. 5 presents a performance comparison of three protocols with EPA and OPA assuming $G_{SR}/G_{RD} = -30$dB. As benchmarks, we include the performance of non-cooperative direct transmission (i.e., no relaying), Alamouti code, and maximal ratio combining (MRC) with two co-located antennas. It should be noted that the inclusion of co-located antenna scenarios help us demonstrate how close the "virtual" antenna implementations can come to their co-located counterparts. The performance of MRC and Alamouti code provide practical lower bounds for Protocol II and Protocol III, which are distributed receive and transmit diversity schemes. To

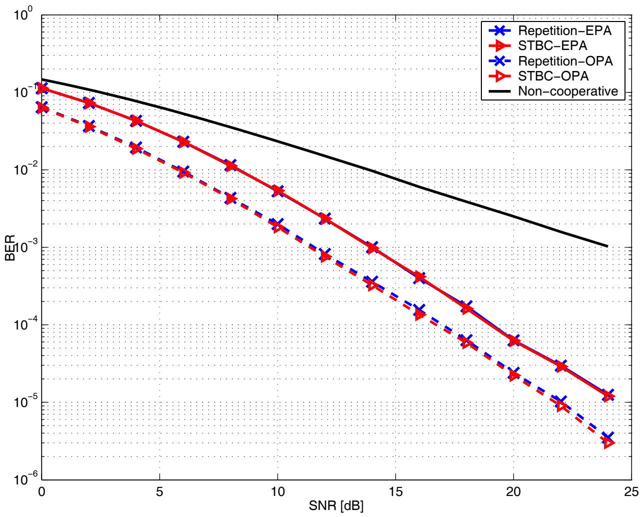

Fig. 6. Performance of Protocol I with repetition and Alamouti codes ($G_{SR}/G_{RD} = -30$dB).

make a fair comparison between cooperative and benchmark schemes which achieve rates of 1/2 and 1 respectively, direct transmission and co-located antenna scenarios are simulated with BPSK. Under EPA assumption, we observe that Protocol I and Protocol II have a similar performance and outperform Protocol III whose diversity is limited to one for the considered $G_{SR}/G_{RD} = -30$dB [8] confirming our earlier observation in (46). Suffering severely from the low SNR in source-to-relay link, Protocol III is even outperformed by direct transmission under the same throughput constraint and is far inferior to its co-located counterpart, i.e., Alamouti scheme. We observe that optimized version of Protocol III achieves a diversity order of two and outperforms direct transmission after $SNR = 8$dB. Unlike Protocol III, Protocols I and II guarantee full diversity under EPA assumption, however their performance is still 3dB away from the MRC performance. Under OPA assumption, Protocol II is able to operate just 0.4dB away from the MRC bound. Another interesting observation from Fig. 5 is that Protocols I and II provide nearly identical performance. It should be emphasized that this observation is made considering the use of Protocol I in conjunction with the underlying Alamouti code. With a more powerful space-time code such as Golden code [27], it is expected that Protocol I will be able to outperform Protocol II providing additional coding gains.

In the following, we discuss the choice of the underlying distributed code (i.e., Alamouti vs. repetition code) for Protocols I and III. As earlier noted, repetition code provides a rate of 1/2 which is the same as distributed implementation of STBC (Alamouti) code for the single-relay scenario under consideration. From the codeword matrix definition given by (16), it can be easily argued that repetition code will not extract spatial diversity under Protocol III. Therefore, STBC is the obvious choice for Protocol III. On the other hand, we observe from Fig. 6 that both repetition code and STBC present a similar performance under EPA for Protocol I. OPA-STBC

[8]We should note that Protocol III under EPA is able to collect a diversity order of two for $G_{SR}/G_{RD} = 0$ and 30dB, but its performance is still inferior to Protocol I and Protocol II.

TABLE II
POWER ALLOCATION PARAMETERS FOR DISTRIBUTED REPETITION CODE UNDER PROTOCOL I.

SNR	G_{SR}/G_{RD}					
	-30dB		0dB		30dB	
[dB]	K_T	K_S	K_T	K_S	K_T	K_S
5	0.9644	0.2035	0.5106	0.4687	0.0346	0.5721
10	0.9643	0.2030	0.5104	0.4597	0.0300	0.5600
15	0.9577	0.5122	0.5445	0.4121	0.0768	0.5219
20	0.9734	0.2847	0.4421	0.4114	0.0681	0.4761
25	0.9734	0.3064	0.5317	0.4112	0.0647	0.4957
30	0.9743	0.2776	0.5475	0.4069	0.1372	0.4598

brings only a small performance improvement over OPA-repetition code[9]. Therefore, both codes can be possibly used in conjunction with Protocol I. We should, however, remind that our discussion here focuses on the single-relay case. For relay network scenarios with more than one relay, the rate loss due to repetition code might exceed that attributable to STBCs [7]. For example, if three relays are available to assist communication then repetition code can achieve a rate of 1/4 while that of G4 [4] is 1/3.

VI. CONCLUSION

In this paper, we have investigated optimum power allocation methods for a single-relay scenario with AaF relaying for Protocols I, II, III of [11]. For each cooperation protocol, we have derived union bounds on BER performance which are then used to optimally allocate the power among cooperating nodes in broadcasting and relaying phases. In comparison to their original counterparts, optimized protocols demonstrate significant performance gains depending on the relay geometry. Our results further provide a detailed comparison among Protocols I, II and III which give insight into the performance of these protocols incorporating the effects of relay location and power allocation.

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