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Note

A note on finite amplitude transverse waves in a thermoelastic solid

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Summary. If thermal and mechanical coupling is neglected, there is a class of isothermal strain energy functions for isotropic compressible hyperelastic solids, that admits the propagation of a finite amplitude transverse wave, without a coupled longitudinal wave, and a subclass that admits the simultaneous propagation of an uncoupled finite amplitude transverse wave and a longitudinal wave [1]. Several examples of this class of strain energy function are discussed. When thermal and mechanical coupling is considered the solid is described as thermoelastic. The purpose of this paper is to investigate the possibility of the propagation of a finite amplitude transverse wave without a coupled longitudinal wave, or the uncoupled simultaneous propagation of a finite amplitude transverse wave and a longitudinal wave, in an isotropic thermoelastic solid that has no underlying deformation. It is shown that an extensive class of isotropic thermoelastic solids does not admit the propagation of an uncoupled finite amplitude transverse wave, with or without an uncoupled longitudinal wave, even if the corresponding hyperelastic solid does.

1 Introduction

If mechanical and thermal coupling is neglected in the analysis of finite amplitude wave propagation in an elastic solid, the constitutive relations for isothermal deformation, that is the isothermal hyperelastic relations, are generally used. This is equivalent to assuming that the solid is piezotropic, that is, mechanical and thermal properties are uncoupled, and the use of isothermal hyperelastic constitutive relations for elastic wave propagation problems is described as the piezotropic approximation. If coupled mechanical and thermal effects are taken into account, the elastic solid is described as thermoelastic. In this paper, we consider thermoelastic constitutive relations that are based on isothermal strain energy functions and the internal energy expressed as a function of deformation and entropy. These relations are used to consider some aspects of the propagation of finite amplitude transverse waves.

The adiabatic approximation implies the neglect of heat conduction and, in the absence of shocks, adiabatic deformation of a thermoelastic solid is isentropic. The isentropic approximation implies the neglect of heat conduction and entropy jumps across any shocks that occur. Thermal conductivities of rubberlike solids that can undergo finite elastic deformation, are several orders of magnitude less than those for metallic materials. Consequently, for finite amplitude wave propagation, in a thermoelastic solid, it is reasonable to neglect heat conduction and assume isentropic deformation, if entropy jumps across shocks are neglected.

It has been shown [1] that, if the piezotropic approximation is assumed, a class of isothermal strain energy functions for isotropic hyperelastic solids admits the propagation of a transverse wave with no coupled longitudinal wave and a subclass admits the simultaneous uncoupled propagation of a transverse wave and a longitudinal wave. In this paper, we investigate the possibility of the propagation of a transverse wave, with or without an uncoupled longitudinal wave, in a class of isotropic thermoelastic solids, whose isentropic strain energy functions are derived from a class of isothermal isotropic strain energy functions, in particular those which admit uncoupled transverse waves.

2 Constitutive equations

For isotropic rubberlike materials, it is physically realistic to consider isothermal strain energy functions of the form:

$$W = \mu\phi_1(I_i) + \kappa\phi_2(J) \tag{1}$$

per unit volume of the natural reference configuration, N_o, at temperature T_o [2]. In Eq. (1), I_i, where $i \in \{1, 2, 3\}$, are the basic invariants of the right or left Cauchy-Green strain tensor, $J = I_3^{1/2}$, μ and κ are the isothermal shear and bulk moduli, respectively, for infinitesimal deformation from N_o. The nondimensional functions ϕ_1 and ϕ_2 vanish for pure dilatation and for isochoric deformation, respectively, and $\phi_2(1) = 0,\ \phi_2'(1) = 0,\ \phi_2''(1) = 1$, where a prime denotes differentiation with respect to the argument. This decomposition has been discussed by Chadwick [2] and others. A subclass of Eq. (1) is

$$W = \frac{\mu}{2}\left[a\left(I_1 - 3I_3^{1/3}\right) + (1-a)\left(\frac{I_2}{I_3} - 3I_3^{1/3}\right)\right] + \kappa\phi_2(J), \tag{2}$$

where $0 < a \leq 1$. The well known Levinson and Burgess [3] and generalized Blatz-Ko [4] strain energy functions can be put in the form of Eq. (2) with

$$\phi_2(J) = \frac{3(1-2v)}{4(1+v)}\left\{3a\left(J^{2/3} - 1\right) + 3(1-a)(J^{-2/3} - 1) + 2(1-2a)(J-1) + \left(2a + \frac{4v-1}{1-2v}\right)(J-1)^2\right\}, \tag{3}$$

and

$$\phi_2(J) = \frac{3(1-2v)}{4(1+v)}\left\{3a\left(J^{2/3} - 1\right) + 3(1-a)(J^{-2/3} - 1) + \frac{(1-2v)}{v}\left[a(J^{-2v/(1-2v)} - 1) + (1-a)(J^{2v/(1-2v)} - 1)\right]\right\}, \tag{4}$$

respectively, where v is Poisson's ratio for infinitesimal deformation from N_o. The functions ϕ_2 given by Eqs. (3) and (4) depend on the ratio $m = \kappa/\mu$ since $v = (3m-2)/(6m+2)$. The Hadamard class of strain energy functions,

$$W = \frac{\mu}{2}[a(I_1 - 3) + (1-a)(I_2 - 3)] + H(I_3), \tag{5}$$

where $0 < a \leq 1$, $H(1) = 0$, $(\mu/2)(2 - a) + H'(1) = 0$, can also be put in the form of a sub class of Eq. (1) as follows:

$$W = \frac{\mu}{2}\left[a\left(I_1 - 3I_3^{1/3}\right) + (1 - a)\left(I_2 - 3I_3^{2/3}\right)\right] + \kappa\phi_2(J), \tag{6}$$

where

$$\phi_2(J) = \frac{H(I_3)}{\kappa} + \frac{3}{2m}\left[a\left(J^{2/3} - 1\right) + (1 - a)\left(J^{4/3} - 1\right)\right]. \tag{7}$$

The strain energy function, for the Hadamard material, does not seem to have appeared in the literature in the form given by Eq. (6).

An empirical form for the function $\phi_2(J)$,

$$\phi_2(J) = \frac{1}{9}\left\{\frac{J^{-9}}{9} + \ln J - \frac{1}{9}\right\}, \tag{8}$$

proposed by Ogden [5], does not depend on m and is in good agreement with experimental data for several rubberlike materials if $0.8 \leq J \leq 1$. Other empirical forms of $\phi_2(J)$ that do not depend on m have been proposed [2]. The function ϕ_2 can be expanded as Taylor series in $(J - 1)$,

$$\phi_2(J) = \frac{1}{2}(J - 1)^2 + O\left((J - 1)^3\right), \tag{9}$$

which suggests the approximation

$$\phi_2(J) = \frac{1}{2}(J - 1)^2 \tag{10}$$

for sufficiently small values of $|J - 1|$. Some caution is necessary in using Eq. (10) when ϕ_2 is given by Eq. (4) since, for values of m that are realistic for solid rubbers, Eq. (10) is a satisfactory approximation for Eq. (4) only if $|J - 1| < m^{-2}$.

A fundamental equation of state for an isotropic thermoelastic solid, in terms of the specific Helmholtz free energy, is of the form:

$$f = \tilde{f}(I_i, T), \quad i \in \{1, 2, 3\}, \tag{11}$$

where T is the temperature, and

$$W = \rho_o \tilde{f}(I_i, T_o), \tag{12}$$

where ρ_o is the density in N_o.

We consider a modified entropic thermoelastic solid, with strictly entropic isochoric deformation and energetic pure dilatation, consequently the specific internal energy, $u(I_i, T)$, is expressible as

$$u = u_1(J) + u_2(T). \tag{13}$$

The specific entropy, $s(I_i, T)$, the specific Helmholtz free energy, f, and the specific internal energy, u, are related by

$$s = -\frac{\partial\tilde{f}}{\partial T} \quad \text{and} \quad u = \tilde{f} + Ts, \tag{14.1, 2}$$

and the specific heat at constant deformation is then given by

$$c = T\frac{\partial s}{\partial T} = -T\frac{\partial^2 \tilde{f}}{\partial T^2} = \frac{du_2}{dT}. \tag{15.1–3}$$

It follows that c is a function of temperature only and the specific entropy is expressible as

$$s = s_1(I_i) + s_2(T). \tag{16}$$

It is assumed that the variation of c with temperature is negligible, for the range of temperature arising in the problems considered, consequently

$$u_2(T) = c(T - T_o). \tag{17}$$

It is convenient to take f, u and s to be zero in N_o, and it then follows from Eqs. (14.2), (15.2) and (17) that

$$f(I_i, T) = \frac{W(I_i)}{\rho_o}\frac{T}{T_o} - u_1(J)\left(\frac{T}{T_o} - 1\right) - cT\ln\left(\frac{T}{T_o}\right) + c(T - T_o). \tag{18}$$

In Eq. (13), u_1 is given by

$$u_1 = \frac{\alpha\kappa T_o}{\rho_o}h(J), \tag{19}$$

where α is the volume coefficient of thermal expansion, and $h(J) = \frac{2}{5}\left(J^{5/2} - 1\right)$ gives satisfactory agreement with experimental data [2]. At this stage, we note that $h(1) = 0$ and $h'(1) = 1$. If $|J - 1| << 1$, $h(J)$ can be approximated by $(J - 1)$. It follows from Eqs. (1), (14.1), (18) and (19) that

$$s_1(I_i) = \frac{\alpha\kappa}{\rho_o}h(J) - \frac{1}{\rho_o T_o}\{\mu\phi_1(I_i) + \kappa\phi_2(J)\}, \quad s_2(T) = c\ln\frac{T}{T_o}, \tag{20}$$

and

$$T = T_o\exp\left(\eta\left[\phi_1(I_i) + \frac{\kappa}{\mu}\phi_2(J) - \varsigma h(J)\right] + \frac{s}{c}\right), \tag{21}$$

where $\eta = \mu/(\rho_o c T_o)$ and $\varsigma = \alpha\kappa T_o/\mu$. It further follows from Eqs. (13), (17), and (19) that the specific internal energy as a function of s and I_i, which is a fundamental equation of state, is given by

$$\hat{u}(I_i, s) = \frac{\alpha\kappa T_o}{\rho_o}h(J) + cT_o\left\{\exp\left(\eta\left[\phi_1(I_i) + \frac{\kappa}{\mu}\phi_2(J) - \varsigma h(J)\right] + \frac{s}{c}\right) - 1\right\}. \tag{22}$$

For isentropic deformation from N_o, with $s = 0$ in N_o, an isentropic strain energy function, $W^* = \rho_o\hat{u}(I_i, 0)$ per unit volume of N_o,

$$W^* = \alpha\kappa T_o h(J) + \frac{\mu}{\eta}\left\{\exp\left(\eta\left[\phi_1(I_i) + \frac{\kappa}{\mu}\phi_2(J) - \varsigma h(J)\right]\right) - 1\right\}, \tag{23}$$

based on Eq. (22), is introduced.

The nominal stress tensor $\mathbf{S}$ is given [5] by

$$\mathbf{S} = 2\frac{\partial\Sigma}{\partial I_1}\mathbf{F}^T + 2\frac{\partial\Sigma}{\partial I_2}(I_1\mathbf{F}^T - \mathbf{C}\mathbf{F}^T) + 2I_3\frac{\partial\Sigma}{\partial I_3}\mathbf{F}^{-1}, \tag{24}$$

where $\Sigma = W$ and $\Sigma = W^*$ for isothermal and isentropic deformation, respectively, from N_o, $\mathbf{F}$ is the deformation gradient tensor, $\mathbf{C} = \mathbf{F}^\mathbf{T}\mathbf{F}$ is the right Cauchy-Green strain tensor, and a superposed T denotes transposition.

3 Wave propagation problem

The deformation for the propagation, in the X_1-direction, of a linearly polarized transverse wave and a longitudinal wave, in an initially undeformed medium is given by,

$$x_1 = X_1 + w_1(X_1, t), \quad x_2 = X_2 + w_2(X_1, t), \quad x_3 = X_3, \tag{25}$$

where $x_i,\ i \in \{1, 2, 3\}$ and $X_\beta, \beta \in \{1, 2, 3\}$ denote the coordinates of a material particle in the spatial configuration and N_o, respectively, referred to the same rectangular Cartesian coordinate system. The components of $\mathbf{F}$ and $\mathbf{C}$, corresponding to Eqs. (25), are given by

$$[\mathbf{F}] = \begin{bmatrix} \delta & 0 & 0 \\ \gamma & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad [\mathbf{C}] = \begin{bmatrix} \delta^2 + \gamma^2 & \gamma & 0 \\ \gamma & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{26.1, 2}$$

where

$$\delta = \frac{\partial x_1}{\partial X_1} = 1 + \frac{\partial w_1}{\partial X_1} \quad \text{and} \quad \gamma = \frac{\partial x_2}{\partial X_1} = \frac{\partial w_2}{\partial X_1}. \tag{27}$$

The basic invariants of $\mathbf{C}$ are

$$I_1 = \mathrm{tr}\mathbf{C},\ I_2 = \frac{1}{2}\left\{(\mathrm{tr}\mathbf{C})^2 - \mathrm{tr}\mathbf{C}^2\right\},\ I_3 = \det[\mathbf{C}],$$

where tr and det denote the trace and determinant, respectively. It follows from Eq. (26.2) that

$$I_1 = 2 + \delta^2 + \gamma^2, \quad I_2 = 1 + 2\delta^2 + \gamma^2, \quad I_3 = \delta^2. \tag{28}$$

Substituting Eqs. (28) into Eqs. (1) and (23) gives

$$W = \hat{W}(\gamma, \delta) \quad \text{and} \quad W^* = \hat{W}^*(\gamma, \delta), \tag{29}$$

respectively. The nominal stress components we need to consider are S_{11} and S_{12}, and these can be obtained as functions of δ and γ from (24) or from

$$S_{11}(\delta, \gamma) = \frac{\partial \hat{\Sigma}(\delta, \gamma)}{\partial \delta}, \quad S_{12}(\delta, \gamma) = \frac{\partial \hat{\Sigma}(\delta, \gamma)}{\partial \gamma}, \tag{30}$$

where $\hat{\Sigma} = \hat{W}$ and $\hat{\Sigma} = \hat{W}^*$ for isothermal deformation and isentropic deformation, respectively, from N_o, and are obtained from Eqs. (1) and (28), and Eqs. (23) and (28), respectively. The nominal stress components S_{11} and S_{12} are also Cauchy stress components for the deformation given by Eqs. (25).

The non-trivial Lagrangian equations of motion are

$$\frac{\partial S_{11}}{\partial X_1} = \rho_o \frac{\partial V_1}{\partial t}, \quad \frac{\partial S_{12}}{\partial X_1} = \rho_o \frac{\partial V_2}{\partial t}, \tag{31}$$

where $V_1 = \partial w_1/\partial t$ and $V_2 = \partial w_2/\partial t$ are the velocity components of a material particle in the X_1- and X_2-directions, respectively, and t is time. Substituting Eqs. (27) and (30) into Eq. (31) gives the system

$$\begin{aligned} c_{11}\frac{\partial^2 w_1}{\partial X_1^2} + c_{12}\frac{\partial^2 w_2}{\partial X_1^2} &= \rho_o \frac{\partial^2 w_1}{\partial t^2}, \\ c_{21}\frac{\partial^2 w_1}{\partial X_1^2} + c_{22}\frac{\partial^2 w_2}{\partial X_1^2} &= \rho_o \frac{\partial^2 w_2}{\partial t^2}, \end{aligned} \tag{32}$$

where

$$c_{11} = \frac{\partial^2 \hat{\Sigma}}{\partial \delta^2}, \quad c_{22} = \frac{\partial^2 \hat{\Sigma}}{\partial \gamma^2}, \quad c_{12} = c_{21} = \frac{\partial^2 \hat{\Sigma}}{\partial \delta \partial \gamma}. \tag{33.1–4}$$

Equations (32) are a hyperbolic system if $c_{11} > 0$, $c_{11}c_{22} - c_{12}^2 > 0$ which implies that $c_{22} > 0$. It follows from Eqs. (32) that uncoupled transverse and longitudinal waves can propagate simultaneously with wave speeds $\sqrt{c_{22}/\rho_o}$and $\sqrt{c_{11}/\rho_o}$, respectively, if

$$c_{11} > 0, \quad c_{22} > 0, \quad c_{12}(\delta, \gamma) = 0. \tag{34.1–3}$$

Also a pure transverse wave can propagate, with wave speed $\sqrt{c_{22}/\rho_o}$, and no longitudinal wave, if

$$c_{22}(1,\gamma) > 0, \quad c_{12}(1,\gamma) = \left(\frac{\partial^2 \hat{\Sigma}(\delta,\gamma)}{\partial \delta \partial \gamma}\right)_{\delta=1} = 0. \tag{35.1–3}$$

If Eqs. (34) are satisfied, then so are Eqs. (35), but the converse is not necessarily true, since (35.2) is a necessary but not sufficient condition for (34.3). An illustration of this is obtained by considering the generalized Hadamard class of isothermal strain energy functions,

$$W = \frac{\mu}{2}[H_1(I_3)(I_1 - 3) + H_2(I_3)(I_2 - 3)] + H_3(I_3), \tag{36}$$

and the piezotropic approximation. In Eq. (35), $(\mu/2)\{H_1(1) + 2H_2(1)\} + H_3'(1) = 0$, $H_3(1) = 0$ and, for compatibility with linear elasticity theory, $H_1(1) + H_2(1) = 1$. If, in addition, $H_1'(1) + H_2'(1) = 0$, Eqs. (35), with $\Sigma = W$, are satisfied and Eq. (36) admits the propagation of a transverse wave with no longitudinal wave, that is with $\delta = 1$. In general, $\frac{\partial^2 W}{\partial \gamma \partial \delta} \neq 0$ if $\delta \neq 1$, however, for an arbitrary value of $\delta > 0$, Eq. (34.3) is satisfied if a further condition $H_1'(I_3) + H_2'(I_3) = 0$ is satisfied. Integrating this condition and noting that $H_1(1) + H_2(1) = 1$ gives an alternative condition, $H_1(I_3) + H_2(I_3) = 1$. Consequently, the uncoupled simultaneous propagation of a pure transverse wave and a longitudinal wave is not admitted by the generalized Hadamard class, unless this additional condition is satisfied. It is worth noting that Eq. (36) can be put in the form (1). Equation (5) satisfies Eq. (34) with $\Sigma = W$ but Eq. (2) does not, unless $a = 1$.

We now consider the isentropic class of strain energy functions given by Eq. (23). This class is appropriate for the study of wave propagation in a non-conducting solid if entropy jumps across shocks are neglected, that is for the isentropic approximation. It is convenient to put Eq. (23) in the nondimensional form:

$$\bar{W}^* = \varsigma h(J) + \frac{1}{\eta}\{\exp(\eta[\bar{W} - \varsigma h(J)]) - 1\}, \tag{37}$$

where $(\bar{W}, \bar{W}^*) = (W, W^*)/\mu$. Propagation of a purely transverse wave with no coupled longitudinal wave is admitted if Eqs. (35.1, 2) with $\Sigma(\delta,\gamma) = W^*(\delta,\gamma)$ are satisfied. Equations (35.1, 2) are given, in nondimensional form, by

$$\frac{\partial^2 \bar{W}^*}{\partial \gamma^2}(1,\gamma) = \exp(\eta \bar{W}(1,\gamma))\left[\eta\left(\frac{\partial \bar{W}}{\partial \gamma}(1,\gamma)\right)^2 + \frac{\partial^2 \bar{W}}{\partial \gamma^2}(1,\gamma)\right] > 0, \tag{38}$$

and

$$\frac{\partial^2 \bar{W}^*}{\partial \delta \partial \gamma}(1,\gamma) = \exp(\eta \bar{W}(1,\gamma))\left[\eta \frac{\partial \bar{W}}{\partial \gamma}(1,\gamma)\left\{\frac{\partial \bar{W}}{\partial \delta}(1,\gamma) - \varsigma\right\} + \frac{\partial^2 \bar{W}}{\partial \delta \partial \gamma}(1,\gamma)\right] = 0, \tag{39}$$

respectively, for Eq. (37) . Since the exponential factor in Eqs. (38) and (39) and the first term in parentheses on the right-hand side of Eq. (38) are nonnegative, it follows that the inequality $\frac{\partial^2 \bar{W}^*}{\partial \gamma^2}(1,\gamma) > 0$ is satisfied if $\frac{\partial^2 \bar{W}}{\partial \gamma^2}(1,\gamma) > 0$. Equation (39) is a necessary condition for the propagation of a transverse wave with no coupled longitudinal wave. This condition cannot be satisfied for the class of isentropic strain energy functions given by (37) with $\bar{W}$ given by the nondimensional form of Eq. (1). In order to show this we consider hyperelastic solids, (i) for which conditions (34) are satisfied, and (ii) for which conditions (35) are satisfied, and then investigate the wave propagation problem for the corresponding thermoelastic solids.

Case (i). Suppose that conditions (34), with $\Sigma = \bar{W}$, are satisfied for a hyperelastic solid and the piezotropic approximation is assumed. The hyperelastic solid then admits the uncoupled simultaneous propagation of transverse and longitudinal waves. This is not true for the corresponding thermoelastic solid with isentropic strain energy function given by Eq. (37), since Eq. (39), which is a necessary condition for the simultaneous propagation, cannot be satisfied unless $\gamma = 0$. This is shown as follows. First note that, for the thermoelastic forms of conditions (35.2) and (34.3), condition (39) is a necessary but not sufficient condition for $\frac{\partial^2 \bar{W}^*}{\partial \delta \partial \gamma}(\delta,\gamma) = 0$, consequently, it need only be shown that $\frac{\partial^2 \bar{W}^*}{\partial \delta \partial \gamma}(1,\gamma) \neq 0$. It follows from Eq. (33.3), with $\hat{\Sigma} = \bar{W}$, and condition (34.3) that $\bar{W} = a_1(\delta) + a_2(\gamma)$, where a_1 and a_2 are twice differentiable functions, and $a_1(1) + a_2(0) = 0$. It then follows from Eqs. (30), with $\Sigma = \bar{W}$, that $a_1'(1) = a_2'(0) = 0$, consequently $\frac{\partial \bar{W}}{\partial \delta}(1,\gamma) = 0$. The condition $\frac{\partial \bar{W}}{\partial \gamma}(1,\gamma) \neq 0,\ \forall \gamma \neq 0$, must be satisfied in general, for a stable elastic solid, otherwise there could be a zero shearing stress for a nonzero value of γ. It then follows from Eq. (39) that $\frac{\partial^2 \bar{W}^*}{\partial \delta \partial \gamma}(1,\gamma) = 0$ if and only if $\gamma = 0$, consequently a transverse wave cannot propagate without a coupled longitudinal wave.

Case (ii). If conditions (35), with $\Sigma = \bar{W}$, are satisfied for a hyperelastic solid and the piezotropic approximation is adopted, the solid admits the uncoupled propagation of a transverse wave, with constant $\delta = 1$. This is not true for the corresponding thermoelastic solid, with isentropic strain energy function given by Eq. (37), since Eq. (39) cannot be satisfied unless $\gamma = 0$. This is shown as follows. Since Eq. (35.2) with $\Sigma = \bar{W}$ is given by $\frac{\partial^2 \bar{W}}{\partial \delta \partial \gamma}(1,\gamma) = 0$ it follows from Eq. (39) that $\frac{\partial^2 \bar{W}^*}{\partial \delta \partial \gamma}(1,\gamma) = 0$ if and only if $\frac{\partial \bar{W}}{\partial \gamma}(1,\gamma) = 0$ or $\frac{\partial \bar{W}}{\partial \delta}(1,\gamma) = \varsigma$. The first of these conditions is valid only for $\gamma = 0$, and the second condition cannot be satisfied for nonzero ς, since it can be deduced from Eqs. (35.2) and (30.1) and $S_{11}(1,0) = 0$ that $\frac{\partial \bar{W}}{\partial \delta}(1,\gamma) = 0$. Consequently, Eq. (39) cannot be satisfied unless $\gamma = 0$, so that a purely transverse wave cannot propagate in the thermoelastic solid.

A hyperelastic solid with the piezotropic approximation does not admit the propagation of an uncoupled purely transverse wave, if $\frac{\partial^2 \bar{W}}{\partial \delta \partial \gamma} \neq 0,\ \forall (\delta,\gamma) \neq (1,0)$. This is also true for the corresponding thermoelastic solid since the term in square brackets in Eq. (39) cannot be 0, except, possibly, for isolated values of γ.

4 Concluding remarks

It has been shown that the thermoelastic model, based on Eqs. (1) and (23), does not admit the propagation of a purely transverse wave, even if the corresponding hyperelastic model with the piezotropic approximation does. This thermoelastic model is strictly entropic for isochoric deformation and energetic for dilatation and is a special case of a more general form of modified entropic elasticity [6] that allows an energetic part of the internal energy in isochoric deformation. Further studies of thermoelastic models based on Eq. (1) and the more general modified entropic elasticity indicate that these models also do not admit the propagation of an uncoupled transverse wave.

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