Mathl Modelling, Vol. 9, No. 8, pp. 651–658, 1987
Printed in Great Britain.
0270-0255/87 $3.00 + 0.00

AN ORTHOTROPIC ELASTIC MODEL FOR RAT ABDOMINAL AORTA

H. DEMIRAY[1,2] and H. A. ERBAY[1]

[1]Department of Applied Mathematics, Tübitak, Research Institute for Basic Sciences, P.O. Box 74, Gebze-Kocaeli, Turkey

[2]Department of Mathematics, Faculty of Sciences, Istanbul University, Vezneciler, Istanbul, Turkey

(*Received March* 1986)

Communicated by X. J. R. Avula

Abstract—Assuming the arterial wall is elastic, homogeneous and incompressible material, an orthotropic strain energy function for rat abdominal artery has been presented. Solving the problem of simultaneous inflation and axial stretch of a cylindrical artery, and then comparing the theoretical results with experiments the numerical values of material coefficients are obtained. Using the least-square error method, within the range of admissible physiological loading, the maximum deviation between the theory and experiment is found to be $<3\%$, which seems to be a fairly good approximation. The variation of circumferential stress and incremental pressure modulus with inner pressure are also reported in the work. Unlike dog abdominal artery, the relation between the incremental modulus and intramural pressure is curvilinear.

1. INTRODUCTION

In many physiological considerations, such as the study of phase velocity in blood vessels, the design of prosthetics and artificial organs, and the investigation of vascular damage under various situations, it is important to know the relation between physiological forces and the resulting deformation (stress–strain relations). It is generally recognized that blood vessel tissue is inhomogeneous, anisotropic, highly deformable and inelastic [1]. In the case of large arteries, however, for deformations not greatly exceeding those occurring *in vivo*, and under the passive state of vascular smooth muscle, the effect of the inelastic component is moderate [2, 3].

In analysing the mechanical response of blood vessels there are two approaches to be followed: the first is the incremental theory which has been widely explored by Kenner [4], Hudetz [5] and Demiray [6]; and the second is the nonlinear stress–strain relation, mainly used by Demiray [7], Vaishnav *et al.* [8], Fung *et al.* [9] and many others in the current literature. In the former approach, as the stress-free state is not defined, it is mathematically impossible to trace the dependence of incremental coefficients on the pressure and the axial force. Although it is not so clear what is meant by a stress-free state for a tissue under *in vivo* conditions, the latter approach proves to be better in relating the elastic stiffnesses to the corresponding physiological forces. Among such researchers, Fung *et al.* [9] are the only ones to have proposed the exponential type of strain energy function; the majority of research workers prefer to use a polynomial type of strain energy function. The drawbacks of a polynomial approximation have been discussed by Fung *et al.* [9].

The present work deals with an orthotropic model for rat abdominal artery, assuming that the arterial wall material is elastic, homogeneous and incompressible. As discussed by Patel and Vaishnav [10], under physiological loadings no shear strains and stresses develop in the wall, thus the strain energy function is taken to be a function of the normal strain components but not of the shearing components. Introducing a strain energy function which comprises both the polynomial and exponential parts, the simultaneous inflation and axial extension of a cylindrical artery is studied under the effects of intramural pressure P and the axial force N. The theoretical pressure and the axial force are compared with the experimental results of Weizsäcker *et al.* [11] and the values of material coefficients obtained. It is seen that the maximum deviation between theory and experiments is found to be 2.5%, which seems to be a good approximation. The incremental pressure modulus and its variation with pressure is also reported in the present work.

2. BASIC EQUATIONS

In this work the arterial wall will be treated as an incompressible, homogeneous and cylindrically orthotropic elastic material. The limitations and validity of these assumptions are given elsewhere [9, 10].

We consider an elastic material subjected to a large deformation which carries a material point $\mathbf{X}$ (X^K; $K = 1, 2, 3$) into a spatial position $\mathbf{x}$ (x^k; $k = 1, 2, 3$) through the deformation $\mathbf{x} = \mathbf{x}(\mathbf{X})$. One of the measures of deformation is described by the Green deformation tensor C_{KL}, defined by

$$C_{KL} = g_{kl} F^k_K F^l_L, \qquad F^k_K \equiv \partial x^k / \partial X^K, \tag{1}$$

where g_{kl} is the metric tensor of the spatial frame x^k and the summation convention applies on the repeated indices. The stress tensor t^{kl} due to the deformation field described above must satisfy the following equilibrium equation:

$$t^{kl}_{;k} = 0; \tag{2}$$

which is due to Cauchy. Here the indices following a semi-colon are used to denote the covariant differentiation of the corresponding tensor field.

For an elastic, homogeneous and incompressible material the stress–strain relations are given by

$$t^{kl} = S g^{kl} + 2 \frac{\partial W}{\partial C_{KL}} F^k_K F^l_L, \tag{3}$$

where g^{kl} is the reciprocal metric tensor of the frame $\mathbf{x}$, W is the strain energy density function and S is the hydrostatic pressure to be determined from the field equations and boundary conditions.

If the functional form of the strain energy density is known, equations (1)–(3), together with the boundary conditions, give sufficient relations to obtain the mechanical field completely. Noting the symmetry properties of physiological loadings and the incompressibility of the material, the functional form of the strain energy density may be described as follows:

$$W = W(C_{22}, C_{33}), \tag{4}$$

where C_{22} and C_{33} are, respectively, the physical components of the Green deformation tensor in the circumferential and longitudinal directions. Thus, from equations (3) and (4), we have

$$t^{kl} = S g^{kl} + 2 \frac{\partial W}{\partial C_{\underline{\Delta\Sigma}}} F^k_{\underline{\Delta}} F^l_{\underline{\Sigma}} \qquad (\Delta, \Sigma = 2, 3). \tag{5}$$

It should be noted here that the summation convention does not apply to repeated but underlined indices.

3. EXTENSION AND INFLATION OF A CYLINDRICAL ARTERY

In this section we will study the simultaneous extension and inflation of a cylindrical shell which is deemed to be a model for an artery. When the arterial segment is subjected to an inner pressure P and the axial force N, the appropriate deformation field in cylindrical polar coordinates is given by

$$r = (R^2/\Lambda_Z + \mathrm{D})^{1/2}, \qquad \theta = \Theta, \qquad z = \Lambda_Z Z, \tag{6}$$

where Λ_Z is the stretch ratio in the axial direction, (R, Θ, Z) and (r, θ, z) are, respectively, the cylindrical polar coordinates of a material point before and after deformation and D is an integration constant to be determined from the boundary conditions. Equations (6) are obtained through use of the incompressibility condition.

Employing equations (1) and (6), the nonvanishing physical components of the Green deformation tensor read as follows:

$$C_{RR} = \frac{1}{(\Lambda_Z \Lambda_\Theta)^2}, \qquad C_{\Theta\Theta} = \Lambda_\Theta^2, \qquad C_{ZZ} = \Lambda_Z^2, \qquad \Lambda_\Theta \equiv \frac{r}{R}, \tag{7}$$

where Λ_Θ is the stretch ratio in the circumferential direction. Then, from equations (5) and (7) the components of the Cauchy stress become

$$t_{rr} = S, \qquad t_{\theta\theta} = S + 2\frac{\partial W}{\partial C_{\Theta\Theta}}\Lambda_\Theta^2, \qquad t_{zz} = S + 2\frac{\partial W}{\partial C_{ZZ}}\Lambda_Z^2. \tag{8}$$

The stress presented in equations (8) should satisfy the equilibrium equation, which is given in cylindrical polar coordinates, as

$$\frac{\mathrm{d}t_{rr}}{\mathrm{d}r} + \frac{1}{r}(t_{rr} - t_{\theta\theta}) = 0. \tag{9}$$

Introducing equations (8) into equation (9) and integrating the result with respect to r, we obtain

$$t_{rr} = \mathrm{C} + 2\int^{r} \frac{\partial W}{\partial C_{\Theta\Theta}} \frac{\Lambda_\Theta^2}{\xi}\,\mathrm{d}\xi, \tag{10}$$

where C is another constant of integration to be determined from the boundary conditions

$$t_{rr} = 0 \text{ at } r = r_o, \tag{11a}$$

$$t_{rr} = -P \text{ at } r = r_i, \tag{11b}$$

where the subscripts o and i stand for the quantities evaluated on the outer and inner surfaces of the cylindrical shell, respectively. If equation (11a) is used in equation (10), the following is obtained:

$$t_{rr} = 2\int_{r_o}^{r} \frac{\partial W}{\partial C_{\Theta\Theta}} \frac{\Lambda_\Theta^2}{r}\,\mathrm{d}r. \tag{12}$$

By changing the integration variable $\xi = \dfrac{r}{R}$ and noting the differential relation

$$\frac{\mathrm{d}r}{r} = \frac{\mathrm{d}\xi}{\xi(1 - \Lambda_Z \xi^2)},$$

equation (12) can be rewritten as follows:

$$t_{rr} = 2\int_{\Lambda_\Theta^o}^{\Lambda_\Theta} \frac{\xi}{1 - \Lambda_Z \xi^2} \frac{\partial W}{\partial C_{\Theta\Theta}}\,\mathrm{d}\xi = S. \tag{13}$$

If the boundary condition on the inner surface of the artery is used, the intraluminal pressure P is obtained as a function of the deformation, i.e.

$$P = 2\int_{\Lambda_\Theta^i}^{\Lambda_\Theta^o} \frac{\xi}{1 - \Lambda_Z \xi^2} \frac{\partial W}{\partial C_{\Theta\Theta}}\,\mathrm{d}\xi. \tag{14}$$

This relation makes it possible to obtain the functional form of the strain energy function by comparing the theoretical pressure with the experimental one.

Noting the relation between the stress components and the hydrostatic pressure (S), from equations (8) and (14) the remaining stress components may be obtained as follows:

$$t_{\theta\theta} = t_{rr} + 2\frac{\partial W}{\partial C_{\Theta\Theta}}\Lambda_\Theta^2, \qquad t_{zz} = t_{rr} + 2\frac{\partial W}{\partial C_{ZZ}}\Lambda_Z^2. \tag{15}$$

The stress component in the axial direction, accordingly the deformation, may be related to the total axial force N through the relation

$$N = 2\pi\int_{r_i}^{r_o} t_{zz}\, r\, \mathrm{d}r. \tag{16}$$

Equations (14) and (16) give the deformation as a function of the pressure P and the axial force N.

4. THIN-WALLED ARTERIES

The formulation presented in the previous section is applicable to arteries whether they are thick- or thin-walled structural elements. For the sake of simplicity in comparing theory with experiments, the arterial wall will be treated as a thin-walled cylindrical shell. To this end, we assume that the thickness h_0 of the arterial wall is small compared to the mean radius $\bar{R}$, i.e. $h_0/\bar{R} \ll 1$. In this case the stress distribution throughout the wall thickness may be taken to be uniform. Noting the following relations:

$$\Lambda_\Theta = \bar{\Lambda}_\Theta \cong \frac{\bar{r}}{\bar{R}}, \qquad \bar{C}_{RR} = \frac{1}{(\Lambda_Z\bar{\Lambda}_\Theta)^2} \cong \frac{h^2}{h_0^2}, \qquad \bar{C}_{\Theta\Theta} \cong \bar{\Lambda}_\Theta^2, \qquad \bar{C}_{ZZ} = \Lambda_Z^2, \tag{17}$$

where h is the deformed value of the wall thickness and $\bar{r}$ is that of the mean radius; we have

$$\left.\begin{aligned} \Lambda_\Theta^{i} &= \bar{\Lambda}_\Theta\left(\frac{1 - h/2\bar{r}}{1 - h_0/2\bar{R}}\right) \cong \bar{\Lambda}_\Theta\left(1 - \frac{h}{2\bar{r}} + \frac{h_0}{2\bar{R}}\right), \\ \Lambda_\Theta^{o} &= \bar{\Lambda}\left(\frac{1 + h/2r}{1 + h_0/2\bar{R}}\right) \cong \bar{\Lambda}_\Theta\left(1 + \frac{h}{2\bar{r}} - \frac{h_0}{2\bar{R}}\right). \end{aligned}\right\} \tag{18}$$

Here $\bar{\Lambda}_\Theta$ is the mean stretch in the circumferential direction. Thus, from equations (13) and (15), the stress components follow:

$$\bar{t}_{\theta\theta} = \bar{t}_{rr} + 2\frac{\partial W}{\partial \bar{C}_{\Theta\Theta}}\bar{\Lambda}_\Theta^2, \qquad \bar{t}_{zz} = \bar{t}_{rr} + 2\frac{\partial W}{\partial \bar{C}_{ZZ}}\Lambda_Z^2. \tag{19}$$

For this particular case, from equation (13), the radial stress may be expressed as

$$\bar{t}_{rr} \cong 2\int_{\Lambda_\Theta^o}^{\Lambda_\Theta} \frac{\partial W}{\partial \bar{C}_{\Theta\Theta}}\frac{\xi}{1 - \Lambda_Z\xi^2}\,\mathrm{d}\xi. \tag{20}$$

Similarly, from equation (14), the approximate inner pressure reads

$$P \cong 2\frac{h_0}{\bar{R}\Lambda_Z}\frac{\partial W}{\partial \bar{C}_{\Theta\Theta}}. \tag{21}$$

When this expression is compared with equation (20) the following relation is found:

$$\bar{t}_{rr} \cong -P/2. \tag{22}$$

This is exactly the same as that proposed by Patel and Vaishnav [10]. Hence, the other stress components become

$$\bar{t}_{\theta\theta} = -P/2 + 2\frac{\partial W}{\partial \bar{C}_{\Theta\Theta}}\bar{\Lambda}_{\Theta}^2, \tag{23a}$$

$$\bar{t}_{zz} = -P/2 + 2\frac{\partial W}{\partial \bar{C}_{ZZ}}\Lambda_Z^2. \tag{23b}$$

The total axial force N acting on the arterial segment is related to the total axial force F measured by the test machine by

$$N = F + \pi\left(\bar{r} - \frac{h}{2}\right)^2 P \cong F + \pi\bar{r}^2 P\left(1 - \frac{h}{\bar{r}}\right) \tag{24}$$

On the other hand, for thin membranes, we have

$$\bar{t}_{zz} = \frac{N}{2\pi\bar{r}h} = \frac{F}{2\pi\bar{r}h} + P\left(\frac{\bar{r}}{h} - 1\right)\Big/2. \tag{25}$$

Combining equations (23b) and (25), the following relation is obtained:

$$\frac{\partial W}{\partial \bar{C}_{ZZ}} = \frac{F}{4\pi\bar{r}h\Lambda_Z^2} + \frac{P}{4h\Lambda_Z^2}. \tag{26}$$

This equation may be used to approximate $\partial W/\partial \bar{C}_{ZZ}$ for various pressures and axial force, whereas relation (21) can be used to assess the functional form of $\partial W/\partial \bar{C}_{\Theta\Theta}$. Having the functional form of these derivatives, one may get some idea about the form of the strain energy function W.

In order to complete the modelling of large arteries we need to know the form of the strain energy density function W. In the current literature there are two avenues to find the form of the strain energy density function. One of these is to express the strain energy function as a polynomial in terms of its arguments [8]. Due to the unstable behaviour of material coefficients under small changes in output data, this approach has not received a great deal of support from researchers in applied mechanics. In the alternative approach, the function W is expressed as an exponential function of the arguments [9]. Fung *et al.*'s proposition gives fairly good results for rabbit arteries but not for rat abdominal aorta. This is probably due to the structural differences in arteries of various animal species..

After examining the experimental results of Weizsäcker *et al.* [11] for rat abdominal artery, we find it very pertinent to propose the following type of strain energy function:

$$W = \alpha_1(C_{\Theta\Theta} - 1)^2 + \alpha_2(C_{ZZ} - 1)^2 + 2\alpha_3(C_{\Theta\Theta} - 1)(C_{ZZ} - 1) + \gamma\exp[\beta_1(C_{\Theta\Theta} - 1)^2 + \beta_2(C_{ZZ} - 1)^2 + 2\beta_3(C_{\Theta\Theta} - 1)(C_{ZZ} - 1)], \tag{27}$$

where α_i, β_i ($i = 1, 2, 3$) and γ are material coefficients to be determined from experimental measurements. Introducing equation (27) into equations (21) and (26), the following relations are obtained:

$$\left.\begin{aligned} P &= \frac{4h_0}{\bar{R}\Lambda_Z}\{\alpha_1(C_{\Theta\Theta} - 1) + \alpha_3(C_{ZZ} - 1) + \gamma[\beta_1(C_{\Theta\Theta} - 1) + \beta_3(C_{ZZ} - 1)]G\}, \\ \frac{F}{8\pi\bar{r}h\Lambda_Z^2} + \frac{P\bar{r}}{8h\Lambda_Z^2} &= \alpha_2(C_{ZZ} - 1) + \alpha_3(C_{\Theta\Theta} - 1) + \gamma[\beta_2(C_{ZZ} - 1) + \beta_3(C_{\Theta\Theta} - 1)]G, \end{aligned}\right\} \tag{28}$$

where the function G is defined by

$$G = \exp[\beta_1(C_{\Theta\Theta} - 1)^2 + \beta_2(C_{ZZ} - 1)^2 + 2\beta_3(C_{\Theta\Theta} - 1)(C_{ZZ} - 1)]. \tag{29}$$

If the variations of pressure and axial force are given for various stretch ratios, equations (28) may be used to determine the numerical values of material coefficients appearing in the strain energy function. This will be done after describing the experimental setup and presenting the results of the measurements.

5. EXPERIMENTS AND RESULTS

The experimental setup, protocol and results employed in the present work are described elsewhere [11]. For self-completeness of the work, the methods are briefly summarized below.

The experiments were performed on segments of abdominal arteries from six adult male rats. In the course of a typical experiment the animal is anaesthetized with pentobarbital, the abdominal is dissected free and cleaned of loose perivascular tissues. A segment about 10 mm in length is removed and placed in a cuvette containing calcium-free EGTA Tyrode solution. After measuring the stress-free dimensions of the segment, its mechanical properties are tested under physiological loading conditions. The segment is cannulated at one end and then inserted in a tensile testing machine, stretched to various longitudinal strain levels and, using a piston pump, inflated at various pressure levels between 0–200 mmHg. In the course of the experiment both the axial force F and the intramural pressure P are measured by transducers, whilst the changes in length and outer diameter are filmed with a 16 mm movie camera. Before starting each measurement the specimen is preconditioned by some 10 loading cycles. After completion of the mechanical testing the stress-free wall thickness h_0 is determined under a microscope on rings cut from the specimen under investigation.

Table 1

P(Pa) Exp.	$10^3 r_o$ (m)	$10^3 r_i$ (m)	$10^3 \bar{r}$ (m)	$\bar{\Lambda}_\Theta$	10^{3F} (N)	P(Pa) Calc.	Deviation (%)
5320	0.761	0.673	0.717	0.972	49.05	5231	−1.6
7980	0.825	0.745	0.785	1.064	48.37	8093	1.4
10,640	0.875	0.800	0.838	1.136	47.10	10,825	1.7
13,300	0.905	0.832	0.869	1.178	45.93	12,961	−2.5
15,960	0.933	0.863	0.898	1.217	43.97	15,938	−0.1
18,620	0.950	0.881	0.916	1.241	42.99	18,853	1.2
21,280	0.960	0.892	0.926	1.255	42.40	21,278	−0.01
23,940	0.968	0.901	0.934	1.266	41.53	23,765	−0.7
26,600	0.975	0.908	0.942	1.276	40.06	26,646	0.1

$\Lambda_Z = 1.58$, $R_o = 0.805 \times 10^{-3}$ m, $R_i = 0.67 \times 10^3$ m, $\bar{R} = 0.738 \times 10^{-3}$ m.

The experimental results measured on rat abdominal aorta are presented in Table 1 for the stretch ratio $\Lambda_Z = 1.58$, which is approximately the *in vivo* prestretch of the vessel under investigation. By utilizing Powell's [12] algorithm, as modified by Zangwill [13], we have determined the material coefficients so as to minimize $(P_T - P_{EX})^2$ and $(F_T - F_{EX})^2$. The results are found to be $\alpha_1 = 31{,}175$ Pa, $\alpha_2 = 9931$ Pa, $\alpha_3 = 8591$ Pa, $\gamma = 40.57$ Pa, $\beta_1 = 4.451$, $\beta_2 = 0.571$ and $\beta_3 = 1.025$. Introducing these numerical values into equations (28) we can calculate the theoretical values of pressure P—listed in the seventh column of Table 1; the deviation between theory and experiment is reported in the last column of the same table.

As may be seen from Table 1, the present model and the experimental results are in quite good agreement. The maximum deviation between the theory and experiment is 2.5%. The variation of inner pressure with the stretch ratio in the circumferential direction and the variation of circumferential stress with inner pressure are shown in Figs 1 and 2, respectively.

Incremental pressure modulus

In vivo blood vessels are generally subjected to a large initial deformation, however this initial deformation changes with varying physiological conditions. Therefore, the classical definition of elastic stiffnesses cannot be used in asserting the stiffness of arterial wall material. As suggested by

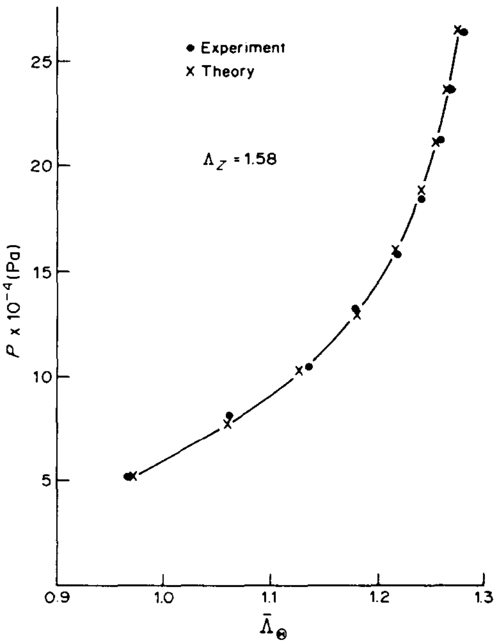

Fig. 1. Variation of inner pressure with circumferential stretch.

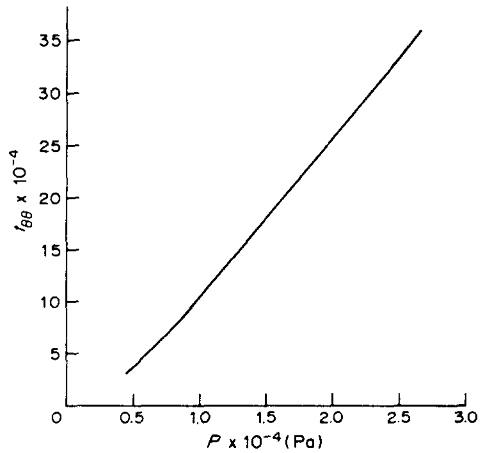

Fig. 2. Variation of circumferential stress with inner pressure.

Bergel [14] and formulated by Demiray [6, 15], the incremental modulus should be employed instead.

The incremental pressure modulus E_p can be obtained from equations (28), by keeping the axial stretch fixed and adding small increments to the inner pressure or mean radius $\bar{r}$. In this case, as a result of the rise in pressure, a small increment $(\Delta\bar{r})$ will develop in the mean radius $\bar{r}$. Using equations (28) and keeping only the linear terms in $(\Delta\bar{r}/\bar{r})$, the following relation between the increments in pressure and the mean pressure is obtained:

$$\Delta P = E_p \frac{\Delta \bar{r}}{\bar{r}}, \tag{30}$$

where the incremental pressure modulus E_p is defined by

$$E_p = \frac{8h_0\bar{\Lambda}_\Theta^2}{\bar{R}\Lambda_Z}\{\alpha_1 + \gamma[\beta_1 + 2(\beta_1[\bar{\Lambda}_\Theta^2 - 1] + \beta_3[\Lambda_Z^2 - 1])^2]G\}. \tag{31}$$

The variation of E_p with inner pressure is shown in Fig. 3. This figure reveals that, unlike dog abdominal aorta [16], the functional relation between the incremental pressure modulus and intramural pressure is curvilinear in the present case.

6. CONCLUSION

Utilizing the large deformation theory of elastic materials, a mathematical model describing the passive mechanical behaviour of rat abdominal aorta is proposed. Keeping in mind the criticisms of the polynomial type of approximation, namely the unstable behaviour of material coefficients, we introduced a strain energy function which contains both the polynomial and exponential parts. By comparing the theoretical results with experiments on rat abdominal artery, values of the material coefficients are obtained through use of the least-square error method. The maximum deviation between the experiments and theory is found to be 2.5%, which is a fairly good approximation. The variation of the incremental pressure modulus with inner pressure is also

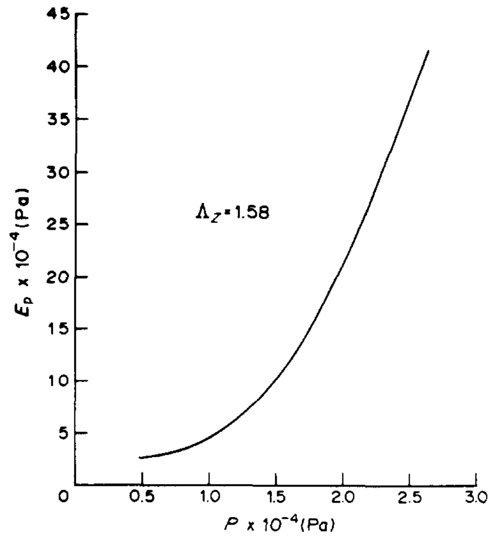

Fig. 3. Variation of incremental pressure modulus with inner pressure.

presented. Unlike dog abdominal aorta, in the present case, the relation between the incremental modulus and inner pressure is curvilinear. This is due to structural differences in arteries of various animal species.

Acknowledgement—We are grateful to Dr H. W. Weizsäcker for providing his experimental data.

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