diversity, and the results are depicted in Fig. 1. In Fig. 2 the ERRP is related to the $m$ parameter for several values of SIR and for $L = 2$ and $L = 3$. Note here that the error performance increases for higher values of the Nakagami $m$ parameter. A slight change in $m$ leads to a significant change in ERRP, especially for large SNR. This occurs because an increase in the $m$ parameter means that the desired signal does not suffer from severe fading, which degrades the error performance. Note also that a higher order of diversity leads to an improvement in the error performance.

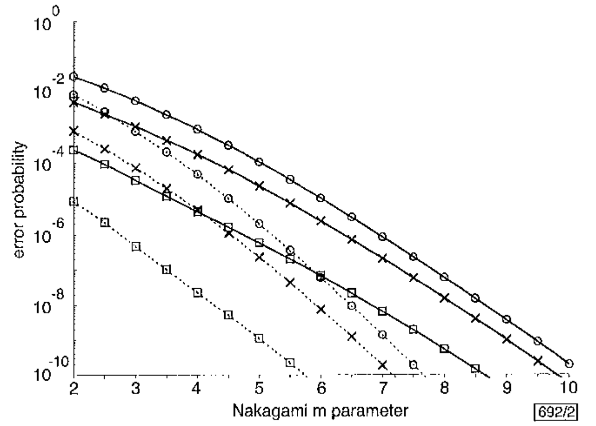


**Fig. 2** *Error probability against Nakagami m parameter for several values of SNR at reference and diversity orders*

—— $L = 2$
- - - - $L = 3$
○ SNR = 0 dB
× SNR = 5 dB
□ SNR = 10 dB

*Conclusions:* A novel simple approach for the evaluation of the error performance in $L$-order equal-gain diversity over Nakagami fading channels has been presented. The obtained closed formulation can be easily used to demonstrate the error performance, helping designers to make decisions on crucial system parameters by taking into account the quality of service (QoS) demands.


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# Performance of space-time coded CDMA systems for wireless communication

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The performance of a space-time coded, synchronous code division multiple access (CDMA) system is investigated over Rayleigh fading channels. An exact pairwise error probability is derived based on which an analytical estimate for bit error probability is computed. The analytical results are verified by computer simulation.

*Introduction:* Space-time coding [1] is a bandwidth and power efficient method of communication over fading channels that realises the benefits of multiple transmit and receive antennas. Performance criteria for space-time codes have been derived in [1] based on the pairwise error probability for both quasi-static flat fading channels and rapid fading channels.

In this Letter, we investigate the performance of space-time coding when it is applied to a synchronous CDMA system operating over rapid Rayleigh fading channels. An exact expression for the pairwise error probability (PEP) for the considered scheme is derived by applying the characteristic function technique in [2, 3], which has been used previously in the performance analysis of trellis-coded modulation (TCM). Our results essentially constitute a generalisation of the technique in [2] for the multiple-transmit and multiple-receive antenna case in a multiuser environment. Based on the new PEP, this Letter also provides analytical bit error performance results for space-time coded systems, contrary to the papers in the literature which only present simulation results.

*System model:* We consider a synchronous space-time coded CDMA system in a multi-user environment with $K$ users. It is assumed that each transmitter is equipped with $M$ antennas and each receiver is equipped with $N$ antennas. For a specific transmitter, data is encoded by the channel encoder and the encoded data is divided into $M$ streams of data by using a serial-to-parallel converter. Each stream of data is then passed through an ideal interleaver. The data streams are then multiplied by the signature sequence $a^i(t)$ assigned to a specific user. At each time slot $t = lT$, the resulting transmitted signal from the $m$th antenna, $m = 1, 2, ..., M$, of the $i$th user, $i = 1, 2, ..., K$ is expressed as

$$s_m^i(t) = \mathrm{Re}\left\{\sqrt{\frac{2}{T}} x_{l,m}^i a^i(t) e^{jw_c t}\right\} \quad (1)$$

where $x_{l,m}^i$ is the complex baseband data signal, $T$ is the signalling interval and $w_c$ is the carrier frequency. The resulting received signal is a noisy superposition of the $M \times K$ transmitted signals corrupted by fading. Assuming that user $k$ is the desired user and that the channel is modelled as Rayleigh fading, the signal received by the $k$th user's antenna $h$, $h = 1, ..., N$, is expressed as

$$r^h(t) = \sum_{i=1}^{K}\sum_{m=1}^{M} \mathrm{Re}\left\{\alpha_{l,m}^{i,h}\sqrt{\frac{2}{T}} x_{l,m}^i a^i(t) e^{jw_c t}\right\} + n^h(t) \quad (2)$$

In eqn. 2, $n^h(t)$ is additive noise modelled as complex Gaussian with double-sided power spectral density $N_0$ and $\alpha_{l,m}^{i,h}$ are statistically independent and identically distributed (iid) complex Gaussian random variables modelling the fading channel from the $i$th user's $m$th transmit antenna to the desired user's $h$th receive antenna; the fading variables are assumed constant during one symbol interval. It should be noted that the fadings from each transmit antenna to any receive antenna are assumed to be independent by placing the antennas sufficiently far apart and symbol-to-symbol independency is realised due to the interleaving-deinterleaving process. The receiver structure consists of a conventional I and Q type demodulator, which is implemented by two correlators matched to the orthonormal carriers, namely $\sqrt{[2/T]}a^k(t)\cos(w_c t)$ and $-\sqrt{[2/T]}a^k(t)\sin(w_c t)$. For the $l$th signalling interval, $l = 1, 2, ..., L$, the output of correlators is given as

$$z_l^h = \sum_{m=1}^{M} \alpha_{l,m}^{k,h} x_{l,m}^k + \sum_{\substack{i=1 \\ i\neq k}}^{K}\sum_{m=1}^{M} R_{k,i}\alpha_{l,m}^{i,h} x_{l,m}^i + n_l^h \quad (3)$$

where $R_{k,i}$ is the cross-correlation between signature sequences

$a^k(t)$ and $a^i(t)$. Since the second term is also complex Gaussian, it can be incorporated into noise term $n_l^h$, resulting in the modified term $\hat{n}_l^h$, including both the effects of multiple access interference and noise. The output is then fed to the space-time decoder which employs the Viterbi algorithm for the minimisation of the metric defined in [1] as

$$\mu(\mathbf{z},\mathbf{x}) = \sum_{l=1}^{L}\sum_{h=1}^{N}\left|z_l^h - \sum_{m=1}^{M}\alpha_{l,m}^{k,h}x_{l,m}^k\right| \qquad (4)$$

under the assumption that ideal channel state information (CSI) is available.

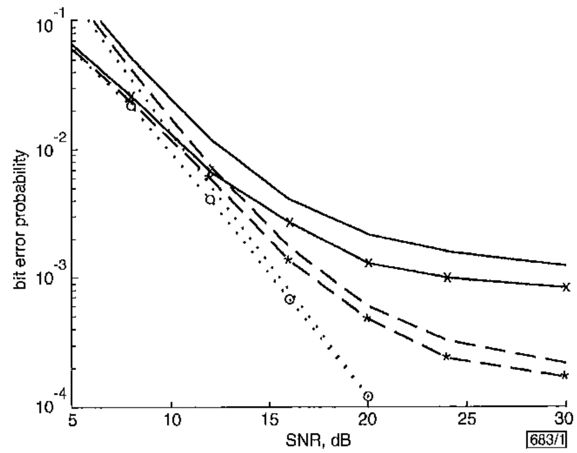


**Fig. 1** *4-PSK four-state space-time code with two transmit antennas and one receive antenna*

——·– analytical, $K = 40$
—×— simulation, $K = 40$
– – – – analytical, $K = 20$
_*_*_ simulation, $K = 20$
········ analytical, $K = 1$
···○··· simulation, $K = 1$

*Performance analysis:* The pairwise error probability $P(\mathbf{x} \rightarrow \hat{\mathbf{x}})$, which represents the probability of choosing the coded sequence $\hat{\mathbf{x}}$ when indeed $\mathbf{x}$ was transmitted, is given by

$$P(\mathbf{x} \rightarrow \hat{\mathbf{x}}) = \Pr[\mu(\mathbf{z},\hat{\mathbf{x}}) \leq \mu(\mathbf{z},\mathbf{x})] = \Pr[D \leq 0] \qquad (5)$$

where for the independent fading case $D$ is

$$\begin{aligned} D = \sum_{l=1}^{L}\sum_{h=1}^{H} D_l^h = \sum_{l=1}^{L}\sum_{h=1}^{H}\Bigg\{ & \left|\sum_{m=1}^{M}\alpha_{l,m}^{k,h}\left(\hat{x}_{l,m}^k - x_{l,m}^k\right)\right|^2 \\ & - \sum_{m=1}^{M}\alpha_{l,m}^{k,h}\left(\hat{x}_{l,m}^k - x_{l,m}^k\right)\left(\hat{n}_l^h\right)^* \\ & - \sum_{m=1}^{M}\left(\alpha_{l,m}^{k,h}\right)^*\left(\hat{x}_{l,m}^k - x_{l,m}^k\right)^*\hat{n}_l^h \Bigg\} \end{aligned}$$

Recalling that $\alpha_{l,m}^{k,h}$ and $\hat{n}_l^h$ are complex Gaussian variables, $D_l^h$ is simply a quadratic form of complex Gaussian variables. Using the characteristic function [2], the pairwise error probability is computed as

$$\begin{aligned} P(\mathbf{x} \rightarrow \hat{\mathbf{x}}) &= \Pr[D \leq 0] \\ &= -\sum \text{Residue}\left[e^{s\delta}\Phi_D(s)/s\right]_{\text{Right plane poles}, \delta=0} \end{aligned} \qquad (6)$$

where $\Phi_D(s)$ is the Laplace transform of the pdf of random variable $D$. Letting $\nu$ denote time instances such that $\hat{x}_{l,m} \neq x_{l,m}$, $\Phi_D(s)$ can be shown to be

$$\begin{aligned} \Phi_D(s) = & \left[\prod_{l\in\nu}\left(\frac{E_s}{4N_0}\sum_{m=1}^{M}|\hat{x}_{l,m} - x_{l,m}|^2\frac{1}{1+M\sum\limits_{\substack{i=1\\i\neq k}}^{K}R_{k,i}^2\frac{E_s}{N_0}}\right)\right]^{-N} \\ & \times\left[\prod_{l\in\nu}\frac{-1}{16(s-p_{1,l})(s-p_{2,l})}\right]^N \end{aligned} \qquad (7)$$

where

$$\begin{bmatrix} p_{1,l} \\ p_{2,l} \end{bmatrix} = \frac{1}{4} \mp \sqrt{\frac{1}{16} + \frac{1}{4\frac{E_s}{N_0}\sum\limits_{m=1}^{M}|\hat{x}_{l,m} - x_{l,m}|^2\frac{1}{1+M\sum\limits_{\substack{i=1\\i\neq k}}^{K}R_{k,i}^2\frac{E_s}{N_0}}}}$$

Setting $K = 1$, the first product term in eqn. 7 is the upper bound derived in [1] for the single user case as expected. In other words, the exact pairwise error probability for the space-time coded CDMA scheme is the upper bound derived in [1] modified by two different correction factors. The first correction factor is given by the second product term, the value of which depends on the poles of the Laplace transform of the pdf of the decision variable. The second correction factor, which demonstrates the effects of multiple access interference, appears in the denominator of the first term and in the expressions for the poles. It should also be noted that setting $K = 1$, $M = 1$ and $N = 1$ in eqn. 7 (i.e. single user case with one transmit and one receive antenna) yields the result in [2] with ideal CSI. An estimation of the bit error probability can be obtained by accounting for error event paths of lengths up to a pre-determined specific value, as in [2]. It should be emphasised that this method does not provide an upper bound on the bit error probability. Therefore, the actual results or the simulation results can be lower or higher than the approximation [3]. As an example, the performance of the four-state space-time code proposed in Fig. 4 of [1] is investigated. Here, the analytical estimates are computed for different number of users, namely $K = 1$, 20 and 40, when two transmit antennas and one receive antenna are employed. Gold sequences of length 63 are used as signature sequences and error events up to length three are taken into account. It is worth noting that considering longer error events results in only a slight improvement in the estimate. The analytical results are plotted in Fig. 1 with the corresponding simulation results. Simulation results demonstrate that the estimates reflect well the general behaviour of the system under consideration, including the error floor introduced by multiple access interference over a broad range of SNRs.

*Conclusion:* An exact PEP has been derived for space-time coded, synchronous CDMA systems over Rayleigh fading channels. Based on the new PEP, analytical bit error performance results have been presented for space-time coded systems in a multiuser environment.


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